~~TOP SECRET~~

EFFECT OF ELLIPTICAL ORBITS ON THE PERFORMANCE OF THE NRL RECONNAISSANCE SATELLITE

INTRODUCTION

The NRL reconnaissance satellite proposal envisions that the satellite will be placed in a substantially circular orbit of mean altitude between 400 and 500 miles. At the present state of the art uncontrolled variations in the launching flight might well cause the satellite to assume a somewhat elliptical orbit. It is therefore necessary to consider the effect of an elliptical orbit on the NRL reconnaissance satellite.

This paper will consider briefly three questions -- the effect of altitude on system performance, the mechanics of an elliptical orbit, and the probable effects of such elliptical orbits on system performance.

THE EFFECT OF ALTITUDE ON SYSTEM PERFORMANCE

Successful operation of the reconnaissance satellite requires that three communications links function -- the intercept link (the path from the enemy transmitter to the satellite intercept receiver), the data link (the path from the satellite data transmitter to the ground-based intercept site), and the command link (the path from the command transmitter to the satellite). There is a further requirement that the intercept and data links operate simultaneously. The requirement on the command link depends on the precise program of the satellite timer. For the data and command links, satisfactory operation means merely that information can be successfully passed over the link. Satisfactory operation of the intercept link not only implies that the satellite can hear the enemy transmission but also that the intercepts are so disposed that an arc of position of the enemy transmitter can be computed.

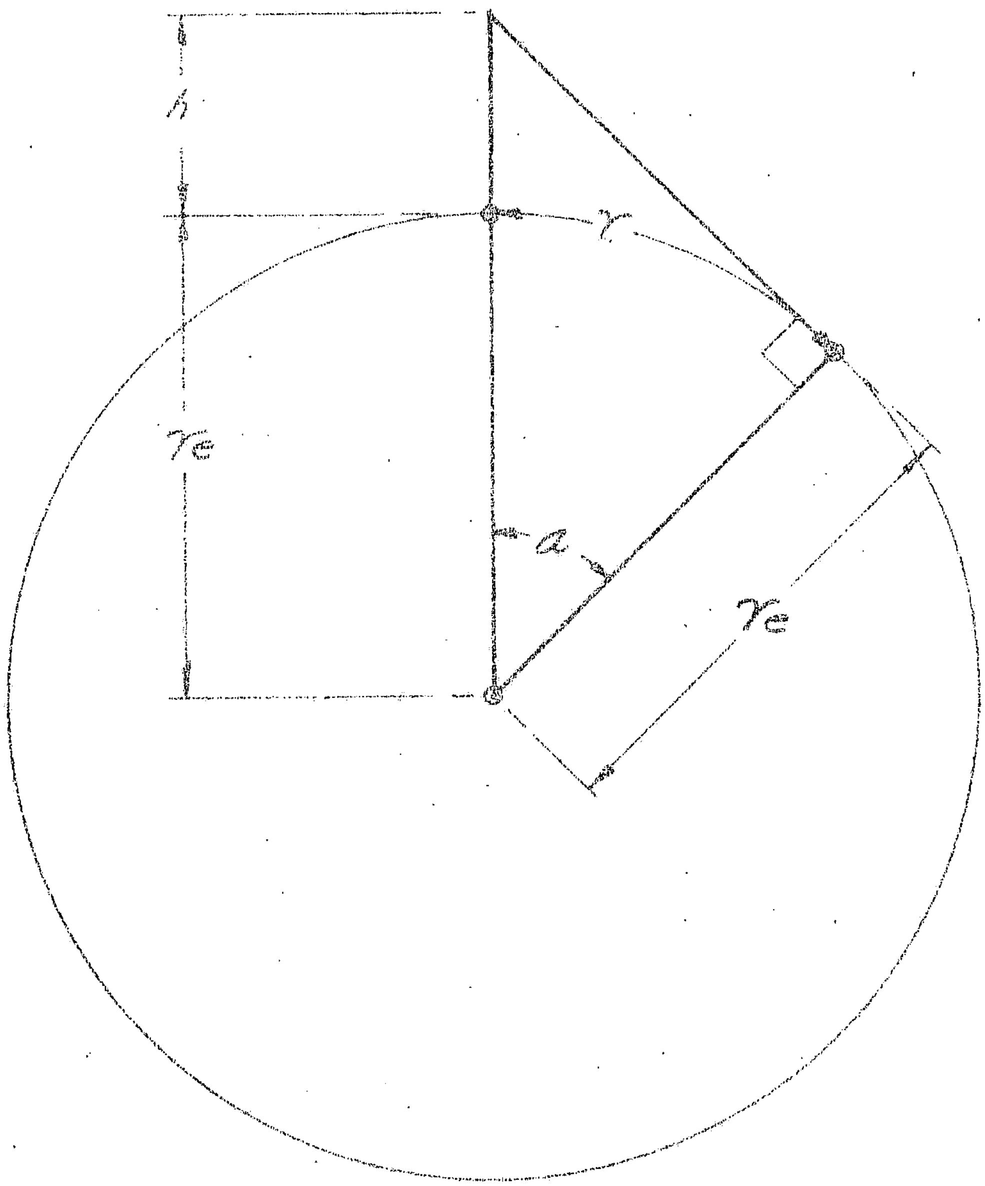
Considering first the command link, it is seen that provided sufficient power is used in the command transmitter the link can be made to function whenever the satellite is in sight of the command transmitter. Referring to Fig. 1 it can be seen that $r = ar_e$ where a is expressed in radians. But $\cos a = r_e / (r_e + h)$. These relations can be used to compute the line-of-sight ground range as a function of altitude. The results of such a computation are given in Table 1.

HANDLE VIA CONTROL SYSTEM ONLY

~~TOP SECRET~~

Approved for Release: 2024/06/08 C05025738

~~TOP SECRET~~ [redacted]



~~TOP SECRET~~ [redacted]

Figure 1

Page 2 of 10 pages

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

~~TOP SECRET~~

Table 1

Altitude (nautical miles)	100	200	400	600	800	1200	1600
Ground Range	820	1150	1580	1910	2150	2530	2820

Considering next the data link, because of the limited transmitted power it might be thought possible that the range of the link would be limited by sensitivity considerations rather than by horizon distances. The following assumptions will be made concerning the data link --

Transmitter Power: 375 mw (the figure of the revised NRL proposal derated by 25% to allow for heating of the satellite.)

Transmitter Antenna: -3 db relative to isotropic

Receiving Antenna: +8 db relative to isotropic

Minimum Usable Signal at Receiver Terminals: -124 dbm

Data Link Frequency: 86 Mc

A simple calculation based on these assumptions reveals a maximum slant range in free space of over 8000 nautical miles. For all practical purposes the limitation on the data path will be simply the horizon distance which has been tabulated in Table 1 above.

In order to estimate the effect of altitude on the performance of the intercept path, calculations will be based on a TOKEN of the following assumed characteristics:

Radiated Power: 600 kw/beam

Antenna Gains (relative to isotropic): Vertical Fan: +39 db (0° to 6° elevation)
+31 db (0° to 25° elevation)

45° Fan: +35 db (0° to 10° elevation)
(i.e. 0° to 7° elevation)
+30 db (10° to 40° slant elevation
i.e. 7° to 28° elevation)

Maximum Side-lobe Antenna Gain: +18 db

Frequency: 3000 Mc

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

~~TOP SECRET~~

he sensitivity of the intercept system be assumed to be -47 dbm which llows for the inferiority of the receiving antenna relative to isotropic.

Calculations based on these assumptions yields the relation etween antenna gain and slant range shown in Table 2.

Table 2

ntenna Gain	+18 db	+30 db	+31 db	+35 db	+39 db
lant Range (Nautical Miles)	187	745	800	1265	2100

The relationship between ground range, slant range, and elevation ngle is depicted in Fig. 2. For any given central angle b measured in idians we have

$$r_g = 3440\ db$$?

$$(r_s)^2 = (3440)^2 + (3440 + h)^2 - 6880\ (3440 + h)\cos b \quad \text{and}$$

$$\cos a = \sin (90^0 - a) = \sin (90^0 + a) =)3440 + h)\sin b / r_s$$

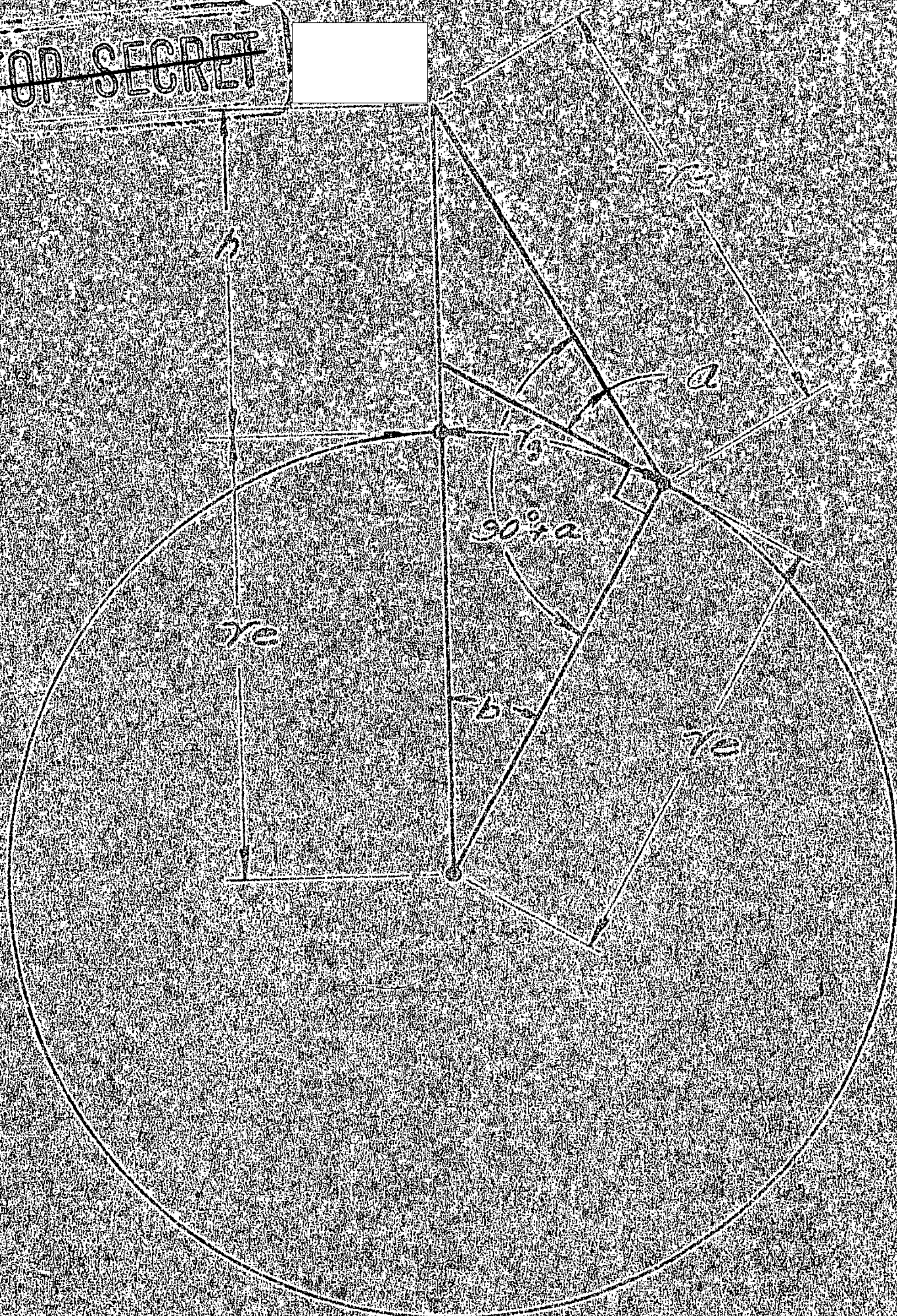
ne results of these calculations are given in Table 3. In the body of this ble, each entry represents the slant range and elevation angle for a given ltitude and central angle. The lower left corner of this table is amputated ecause of the horizon limitation.

If the data of Tables 2 and 3 are combined, it is possible to determine what ground ranges intercepts are possible for various altitudes. This formation is given in the upper half of Table 4. The notations "vertical" nd "45°" refer to the two fans of the TOKEN and the letters "U" and "L" dicates that the upper and lower beams respectively can be intercepted nder the given conditions. An entry in parenthesis indicates that interception ay possibly occur. Inspection of the upper half of Table 4 would seem to dicate that the optimum satellite altitude is in the range of 400 to 600 miles. these altitudes side lobes will not be heard (since their interception range only 187 miles) and interception of the main beams would occur in an nnulus of approximately 250 mile thickness at a distance of 1300 to 1700 iles from the sub-satellite point depending on the exact altitude. Altitudes

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

~~TOP SECRET~~

~~TOP SECRET~~ [redacted]



~~TOP SECRET~~ [redacted] FIGURE 2

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

~~SECRET~~ ~~TOP SECRET~~

Table 3

Central Angle (Radians)	Ground Range (Naut. Miles)	Altitude (Nautical Miles) 100		200		400		600		800		1200		1600	
0.00	0	100	90°	200	90°	400	90°	600	90°	800	90°	1200	90°	1600	90°
0.05	172	202	28°	267	47°	440	64°	628	71°	824	75°	1216	79°	1614	[illegible]
0.10	344	363	13°	407	27°	541	45°	707	55°	882	57°	1265	69°	1654	72°
0.15	516	532	6°	567	16°	676	32°	820	43°	984	50°	1341	59°	1718	[illegible]
0.20	688	704	3°	734	10°	829	23°	956	33°	1110	41°	1441	50°	1804	[illegible]
0.25	860	(876)		905	6°	991	17°	1107	26°	1244	32°	1552	42°	190[illegible]	[illegible]
0.30	1032			1076	4°	1158	11°	1266	20°	1395	26°	1694	36°	2029	[illegible]
0.35	1204			(1249)		1328	7°	1430	14°	1553	21°	1837	30°	2161	[illegible]
0.40	1376					1499	4°	1599	10°	1716	16°	1990	25°	2300	[illegible]
0.45	1548					1672	2°	1770	7°	1884	12°	2150	20°	2453	[illegible]
0.50	1720							1941	4°	2065	10°	2311	16°	2616	[illegible]
0.55	1892							2114	3°	2223	5°	2480	12°	2770	[illegible]
0.60	2064									2397	3°	2650	9°	29[illegible]	[illegible]
0.65	2236									(2568)		2820	5°	3106	[illegible]
0.70	2408											2991	2°	3277	[illegible]
0.75	2580													3444	4°
0.80	2752													3616	[illegible]

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

~~TOP SECRET~~ ~~SECRET~~

~~TOP SECRET~~

Table 4

Altitude (Nautical Miles)		Ground Range (Nautical Miles) 172	344	516	688	860	1032	1204	1376	1548	1720	1892	2064	2236	2408	2580	2752
100	Vertical	(U)	U	U-L	L												
	45°	U	U	U-L	L												
200	Vertical		(U)	U	U	L	L										
	45°		U	U	U	L	L										
400	Vertical				(U)			(L)	L	L							
	45°				(U)			(L)									
600	Vertical									L	L	(L)					
	45°																
800	Vertical											(L)					
	45°																
1200	Vertical																
	45°																
1600	Vertical																
	45°																
100	Vertical	(U)	U	U-L	L												
	45°	U	U	U-L	L												
200	Vertical		(U)	U	U	L	L										
	45°		U	U	U	L	L										
400	Vertical				U	U	(U)	(L)	L	L							
	45°				U	U		L	L	L							
600	Vertical					(U)				(L)	L	L					
	45°					(U)				(L)							
800	Vertical											L	L				
	45°																
1200	Vertical												(L)	L	(L)		
	45°																
1600	Vertical																
	45°																

~~TOP SECRET~~

~~SECRET~~

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

in excess of 700 miles would be undesirable because of the slant range limitation.

If the sensitivity of the intercept receiver could be increased from -47 dbm to -50 dbm, Table 2 would be replaced by Table 5 and the

Table 5

Antenna Gain	+18 db	+30 db	+31 db	+35 db	+39 db	
Slant Range	265	1055	1130	1780	2970	Nautical Miles

lower half of Table 4 would apply. In this event altitudes from 400 to 1200 nautical miles would be satisfactory, although altitudes below 600 miles would give an uncomfortably thick annulus. Another advantage of this mode of operation would be the greater horizon distance for the operation of the command and data paths.

ORBIT MECHANICS

If the oblateness of the Earth, air friction, and the perturbations of the Sun and Moon are neglected, the satellite will travel in an ellipse one of whose foci is at the center of the Earth. The orbit may be described by stating the length of the axes, the inclination with respect to the Earth's polar axis, and the position of perigee with respect to the celestial sphere.

There is only one unique orbit passing through a given pair of points in coordinate and momentum space. Thus if both the position and velocity vectors are known at the moment of satellite separation, the orbit can be predicted. Conversely, to place the satellite in some desired orbit it is necessary to bring the satellite to some point on that orbit and impart to it the velocity characteristic of that point in the orbit.

If a circular orbit of given radius and inclination is desired, it is necessary that at the end of powered flight the satellite be at the desired altitude, have zero velocity normal to the Earth, and have the tangental velocity characteristic of that altitude. It might well be instructive to hypothesize a given orbit and calculate the effect of small errors in velocity or orientation. Only the results will be given here, the details of the computation may be found in the appendix.

Assume that it is desired to place the satellite in a circular orbit at an altitude of 560 miles. This implies that at separation the satellite is at an altitude of 560 miles and travelling horizontal to the surface of the Earth at slightly less than 4 miles per second.

~~P SECRET~~

If the velocity at separation is 2% high, the satellite will assume elliptical orbit whose altitude varies from 560 to 880 miles. If the velocity separation is 2% low, the satellite will assume an elliptical orbit whose itude varies from 560 to 240 miles. If the altitude at separation is 616 miles % high) the altitude will vary between 616 and 738 miles. If the altitude at eration is 504 miles (10% low), the altitude will vary between 398 and 504 miles. ally if the velocity and altitude are correct, but the velocity is directed 2° her towards or away from the Earth the altitude will vary between 424 and 696 miles.

If the reconnaissance gear were designed for a 560 mile altitude, any hese orbits would probably be satisfactory with the exception of the low ocity orbit. Ultimately, those responsible for the vehicle design must decide hese limits can be met. It should be pointed out that the Soviet 1957-alpha was ced in an almost perfect circular orbit. On the other hand, some of the American ellites were placed in highly eccentric orbits. It is not known to what extent ularity was a goal in the programming of these launchings, although from the nt of view of a geophysicist an eccentric orbit is desirable as its permits sampling wide range of altitudes.

If an elliptical orbit is established because of an error in velocity, the ellite will still be at design altitude once every period at a point corresponding he separation point. For a satellite launched from the Northern sub-tropics 70° orbit, this point, be it perigee or apogee, will lie in the northern perate or sub-arctic latitudes. Because of the oblateness of the Earth and turbations by other bodies the orbit will process substantially in its own plane.

The life of the satellite would be reduced if its perigee were at a ficiently low altitude that significant air friction would be experienced.

E EFFECT OF ELLIPTICAL ORBITS ON SYSTEM PERFORMANCE

If the satellite is equipped with a -47 dbm intercept receiver, and ptical orbit whose altitude varied between 350 and 650 nautical miles would serious affect the performance of the intercept link, although the radius of intercept annulus would vary radically. If a good orbit ephemeris were ilable, this should not seriously complicate the interpretation of the received a. A 350 mile perigee altitude might limit system performance by reducing the ge of the data link to below 1500 miles.

If the satellite is equipped with a -50 dbm receiver and orbit whose tude varied between 350 and 1200 miles would be acceptable, although a her perigee altitude would be preferred. At the higher altitudes the command data links would present no problems.

HANDLE VIA

~~TOP SECRET~~

If the orbit were still more eccentric the initial value of the satellite would not be greatly reduced since the altitude will be approximately correct when the satellite passes over the Soviet Union on a northerly course. A highly eccentric orbit might limit the useful life in two ways -- a low perigee would cause premature disintegration of the satellite, and eventually the orbit would process and the satellite would no longer be at design altitude when it passed over the Soviet Union. The exact rate of precession is a difficult calculation -- a rough guess is that it would not become serious for several months by which time the satellite may well have served its purpose.

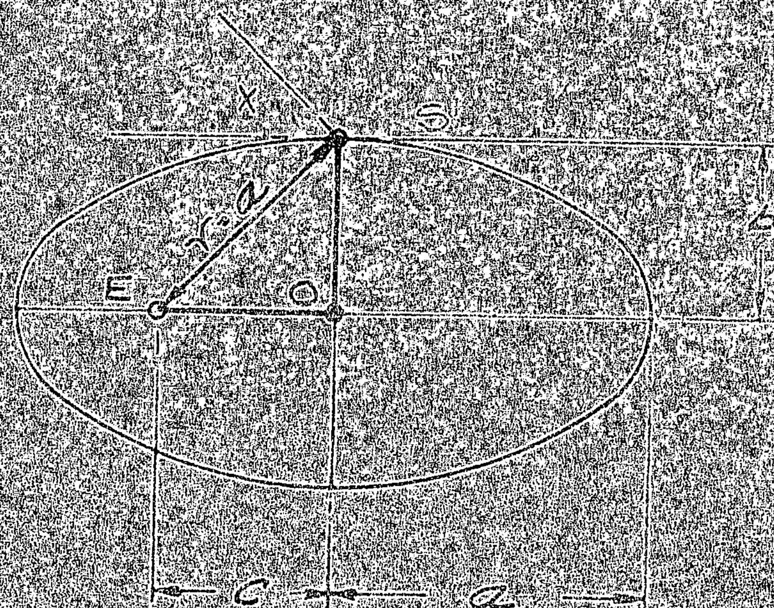
CONCLUSIONS

The proposed NRL reconnaissance satellite will perform its designed function over about a 2:1 range in altitude. For this reason, moderately eccentric orbits should prove no handicap provided an accurate emphemeris is available to the organization processing the data. The optimum altitude for the proposed satellite is in the vicinity of 500 miles, the maximum altitude for TOKEN interception is somewhat below 800 miles, the minimum altitude for reception of intercepts from the central part of the Soviet Union and retransmission to our intercept sites is in the vicinity of 400 miles.

A slight increase in the sensitivity of the intercept receiver would increase still further the range of altitudes for satisfactory operation. In this event the optimum altitude would be about 800 miles, the maximum altitude for TOKEN interception above 1200 miles, and the minimum altitude again in the vicinity of 400 miles.

Even if the parties launching the satellite cannot guarantee meeting these loose specifications it should still be possible to achieve a reasonably satisfactory orbit. Assuming that final velocity is the hardest parameter to control, the launching would be programmed for a final velocity equal to the sum of the design velocity and the probable error in final velocity. This would yield a reasonably high probability that the perigee of the orbit would be at design altitude assuring a long life for the satellite. Although apogee might be considerably higher than design altitude it would occur in the Southern hemisphere and northward passes over the Soviet Union should occur at very near design altitude for a period starting immediately after launching and only ending weeks or months later when the orbit has processed considerably.

~~TOP SECRET~~ [redacted]



~~TOP SECRET~~ [redacted]IGURE 3

HANDLE VIA
BYEMAN
CONTROL SYSTEM ONLY

APPENDIX

An object in an elliptic orbit of semi-major axis, a, at a radius, from the center of attraction has a velocity given by

$$v^2 = u\left(\frac{2}{r} - \frac{1}{a}\right)$$

ere u is a constant describing the strength of the gravitational field. For ircular orbit $r = a$ and $v^2 = u/r$. The first equation may be solved for ielding

$$a = \frac{r}{2 - rv^2/u}.$$

fferentiating with respect to v

$$da/dv = \frac{2r^2 v/u}{(2 - rv^2/u)^2} = \frac{2(r/v)(rv^2/u)}{(2 - rv^2/u)}$$

a nearly circular orbit the following approximations can be made

$/u \approx 1 \qquad r = a$ yielding

$/dv = 2a/v$ or $da/a = 2dv/v$.

refore if r is 4000 miles (altitude = 560 miles) a 2% error in v will cause % (160 mile) error in a with the signs of the errors corresponding. But since separation point must be either perigee or apogee since ex hypothesi the h is normal to the surface of the Earth the total variation in orbit radius and refore altitude will be 320 miles.

Now differentiating the expression for a with respect to r

$$/dr = \frac{1}{2 - rv^2/u} + \frac{rv^2/u}{(2 - rv^2/u)^2} = 2$$

s if r at separation is 4056 miles a will be 4112 miles and apogee will be 78 miles. If r at separation is 3944 miles, a will be 3944 miles and perigee l be 3832 miles.

In the case where the velocity is correct in magnitude but wrongly ented r = a = 4000 but the separation point can be neither a perigee nor apogee since the path is not parallel to the surface of the Earth. Referring to Fig. 3 the Earth is seen to be displaced from the center of the ellipse by a distance c. The separation point must lie somewhere on the ellipse at a distance r = a from E. However, for any ellipse

$$a^2 = c^2 + b^2$$

where b is the semi-minor axis. Therefore the separation point S must lie on the intersection of the semi-minor axis and the ellipse. Now the launching angle error, x, is the angle between the tangent to the ellipse at S and the normal to the line ES. By the law of corresponding perpendiculars this is equal to the angle ESO whose sine is c/a. It therefore follows that c = a sin x, or if a = 4000 and x = 2°, c = 136 perigee and apogee are at 3864 and 4136 miles respectively.