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Subject: Probability of Intercept by P-989 Telemetry Collection System

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An evaluation has been requested of the statistical nature of the telemetry samples available to a P-989 satellite in a 275 NM circular, high inclination orbit. The primary assumptions for the study are that the telemetry signal originates at 45°N latitude, is active for a specified period of time once per month, and is intercepted when it is active and above the P-989 satellite 0° horizon. The probability of obtaining more than a given number of minutes of continuous intercept per active period is plotted as a function of total signal on time. Cumulative probability curves for many active periods are also given.

Because the statistical study applies to any single signal of interest (SOI) rather than being restricted to telemetry, the signal is referred to in the report simply as the SOI.

The report is organized into three parts:

1. Description of the model
2. Summary, probability curves, and conclusions
3. Description of the method of analysis

<u>Model Used for Study</u>

It is assumed that the signal of interest (SOI) originates at 45°N latitude and is turned on once per month at a random time with respect to the P-989 orbital position and stays on for a fixed length of time. This SOI on-time is treated as a parameter in the following study.

It is assumed that the P-989 satellite is in a 275 NM circular orbit at 80° inclination and that it will collect the SOI if it is active and above the P-989 0° horizon.

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The parameter of primary interest is the probability with which the P-989 system will obtain meaningful samples of the SOI. Meaningful is defined here by treating the time span of the SOI sample as a separate parameter from SOI on-time. The reader can decide for himself what meaning to apply to a 5 minute sample of a signal which was on for an hour (for example).

A second parameter of interest is the cumulative probability of obtaining a meaningful sample of SOI as a function of the number of times it turns on.

Other parameters of interest are the sensitivities of the above results to SOI latitude change, orbital inclination change, the use of more than one P-989 system, and to more frequent SOI turn-ons. Another sensitivity which is not given here is the sensitivity to P-989 orbital altitude. Over the 250-500 NM altitude range readily available to P-989, this sensitivity is small. Also, the rapid increase in environmental interference with P-989 altitude negates the geometrical advantage of increased SOI contact time at higher orbital altitudes.

Summary and Conclusions

For the above emitter model the probability of fleeting contact (appearance of the active SOI above the P-989 0° horizon) is plotted as a function of SOI on time for SOI latitudes of 25°N, 45°N, 65°N, and 78°N. As shown, the probability model changes as the SOI on-time passes through the 95 minute orbital period. For an SOI latitude greater than 78°N, contact will occur on every orbit and the probability will be unity for on-times which exceed one orbital period minus the visibility time. For any SOI latitude, the probability will reach unity for an on-time of less than 12 hours (the time for interchange of the ascending and descending nodal longitudes of the orbit).

For the 45°N SOI, Figure 2 shows the probability of fleeting contact and the probabilities of contacts exceeding 1, 3, 5, 8, 10, and 11 minutes, respectively. These contact times include the time required to recognize and lock to the SOI frequency (typically 10 seconds for the SAVANT P-989 telemetry system). Since the maximum horizon-to-horizon visibility time to a 275 NM orbit is 11.6 minutes, the probability of one contact exceeding 11.6 minutes is zero regardless of SOI on-time. The locus of starting points of these curves, shown dashed, is the probability of the SOI on-time falling entirely within the P-989 contact time. This curve also gives the probability of obtaining any specified (as opposed to arbitrary) time segment of the SOI, regardless of SOI on-time.

Figures 1 and 2 give intercept probabilities per SOI on-time. Figure 3 gives the cumulative probability of intercept as a function of the single on-time probability and of the number of on-times. Because the meaningful parameter in Figure 3 is number of on-times rather than time between on-times, the abscissa is months, if the SOI is on once per month; weeks, if the SOI is on once per week, etc.

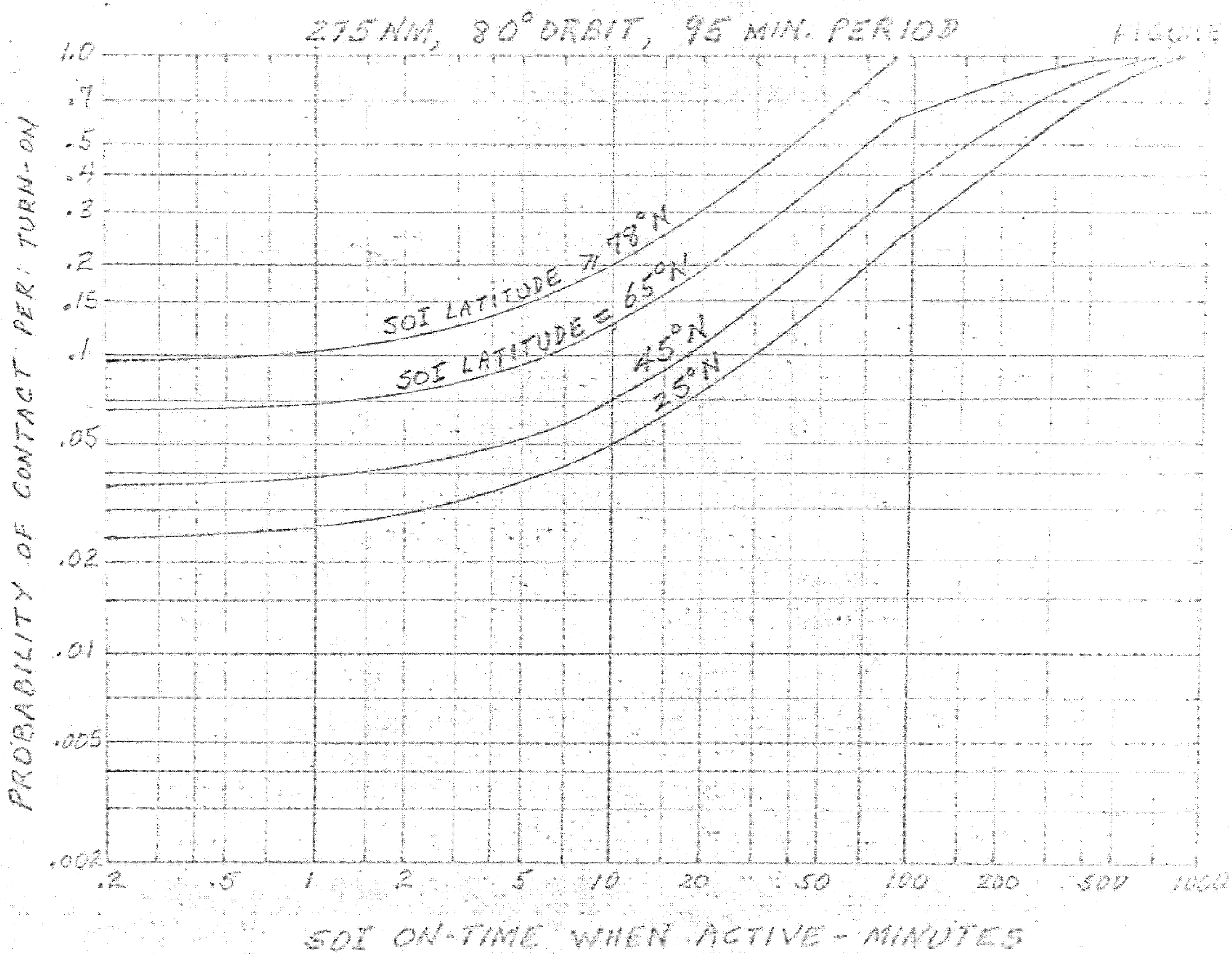

FIGURE 1. PROBABILITY OF FLEETING CONTACT

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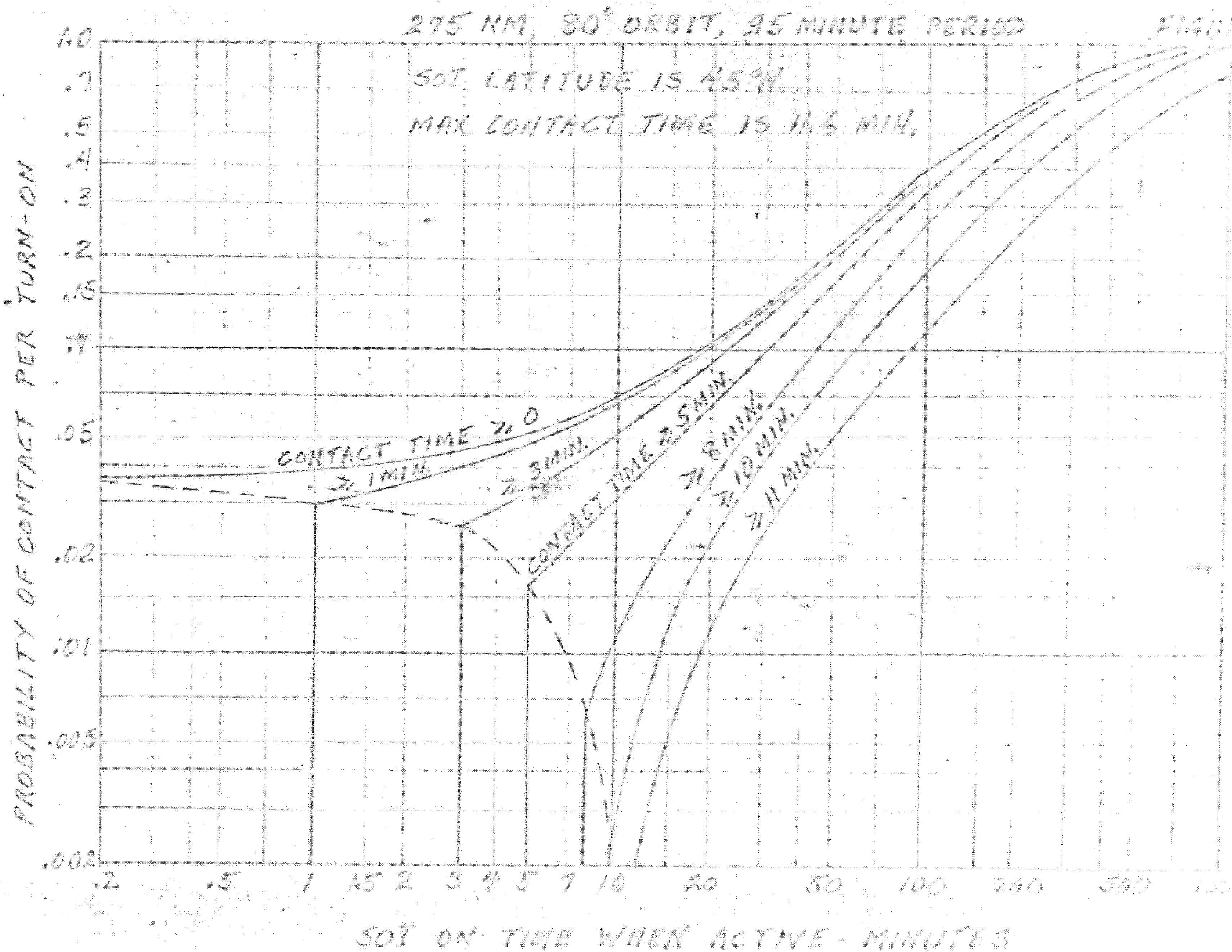

FIGURE 2. PROBABILITY OF MEANINGFUL CONTACT

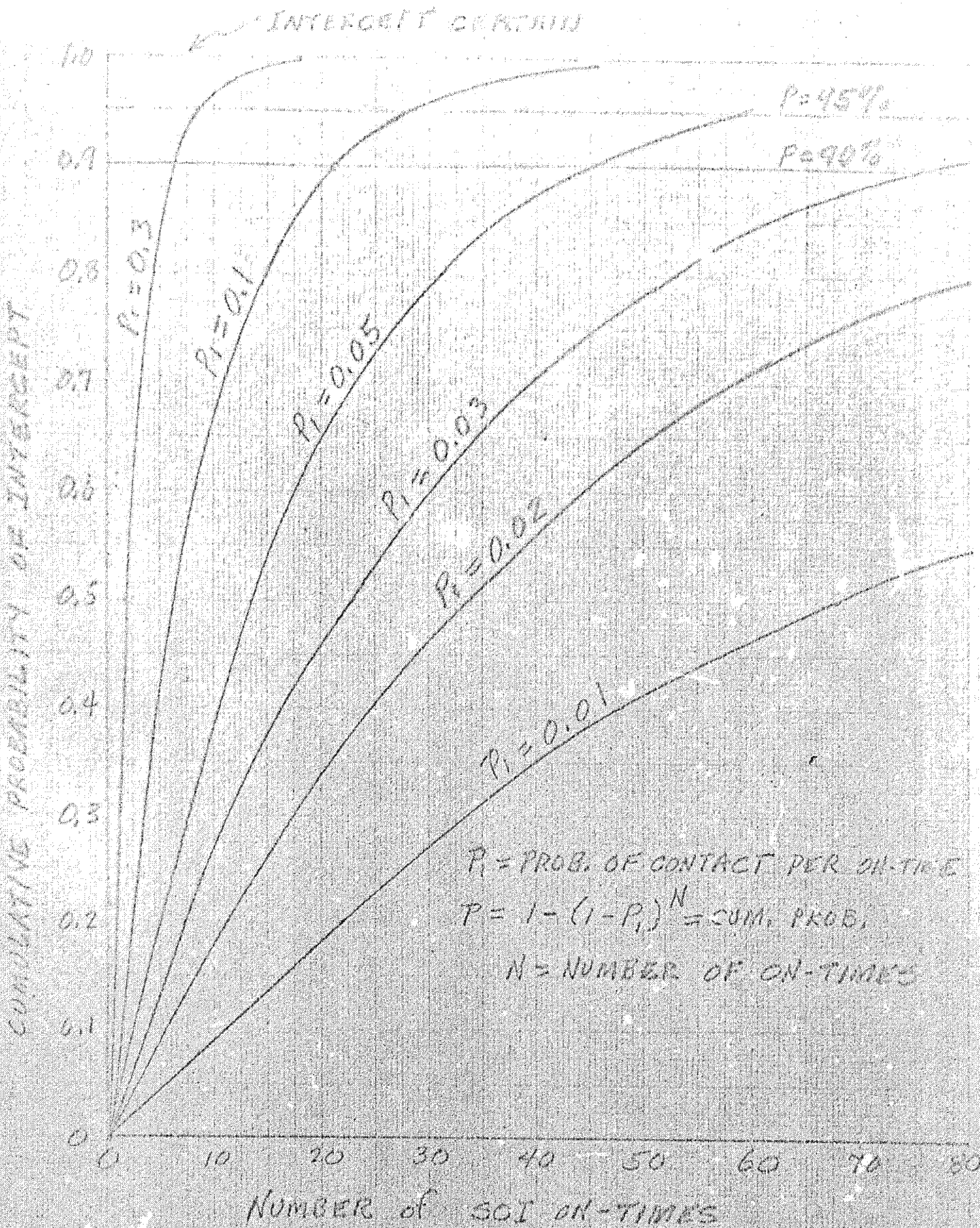

FIGURE 3. CUMULATIVE PROBABILITY OF INTERCEPT

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As an example of the use of the curves, let us compute the number of SOI on-times (months) to reach a 50% probability of obtaining a steady 5 minute sample from a 45°N SOI which turns on once per month for 15 minutes. From Figure 2 the probability per turn-on of obtaining a 5 minute sample is .05 From Figure 3 the number of turn-ons (months) to obtain 50% probability is 13.5. However, the probability per turn-on of obtaining a specified 5 minute sample is .016, and the number of months to obtain this specified sample with 50% probability is roughly 45.

We will now discuss the sensitivity of the above results to other parameters. The sensitivity to SOI latitude is given in Figure 1. For a given SOI on-time, the probability of contact increases almost linearly with the number of passes per day above the SOI horizon. Passes per day as a function of SOI latitude are given in Figure 4 for orbital inclinations of 60°, 70°, 80°, and 90°. If the SOI latitude is more than 82°N, it cannot be seen from a 60° orbit. By contrast the higher inclination orbits start obtaining contacts across the pole as the SOI latitude increases. The discontinuity in the shape of the curves occurs at that latitude where the orbit plane can no longer pass completely north of the SOI horizon. The orbit inclination which maximizes contacts per day with an SOI at latitude ϕ is ϕ +22° for the 22° earth central angle to the horizon from a 275 NM orbit until ϕ reaches 68°. For $\phi \geq 68°$ a 90° (polar) orbit provides the best coverage.

The sensitivity of the probabilities to more frequent SOI turn-ons was previously discussed in explaining Figure 3. The cumulative probability of intercept is a direct function of the number of turn-ons rather than the time between turn-ons. Hence, the assumed frequency of turn-on must be used to convert the abscissa of Figure 3 to time. By contrast, increasing the number of P-989 systems will increase the probabilities of contact per SOI turn-on of Figures 1 and 2. If the P-989 orbits are properly phased, then a linear increase will occur. With two P-989 systems, each probability will be doubled; with three, each will be tripled (for SOI on-times up to that which produces unity probability). If the P-989 systems are not phased, then additional systems will provide a less than proportional increase in contact probability, but only slightly less for a small number of additional systems. Roughly 30 phased systems are required for continuous coverage.

A final parameter which is shown in Figure 5 is the per cent on-time represented by the few minutes per month SOI activity.

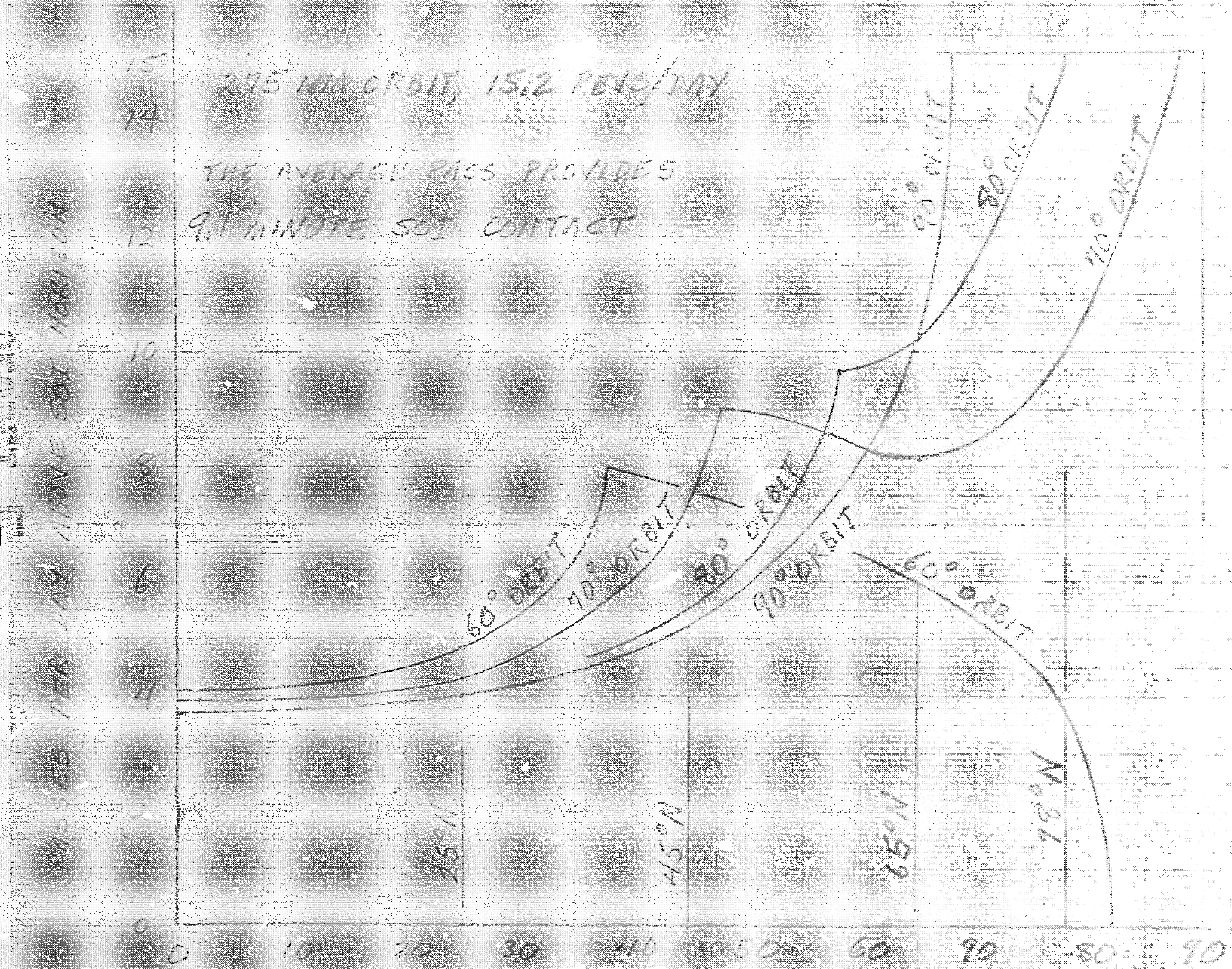

275 NM ORBIT, 15.2 REVS/DAY
THE AVERAGE PASS PROVIDES
9.1 MINUTE SOI CONTACT
PASSES PER DAY ABOVE SOI HORIZON
SOI LATITUDE, DEGREES
0
2
4
6
8
10
12
14
15
0
10
20
30
40
50
60
70
80
90
60° ORBIT
70° ORBIT
80° ORBIT
90° ORBIT
25°N
45°N
65°N
78°N

FIGURE 4

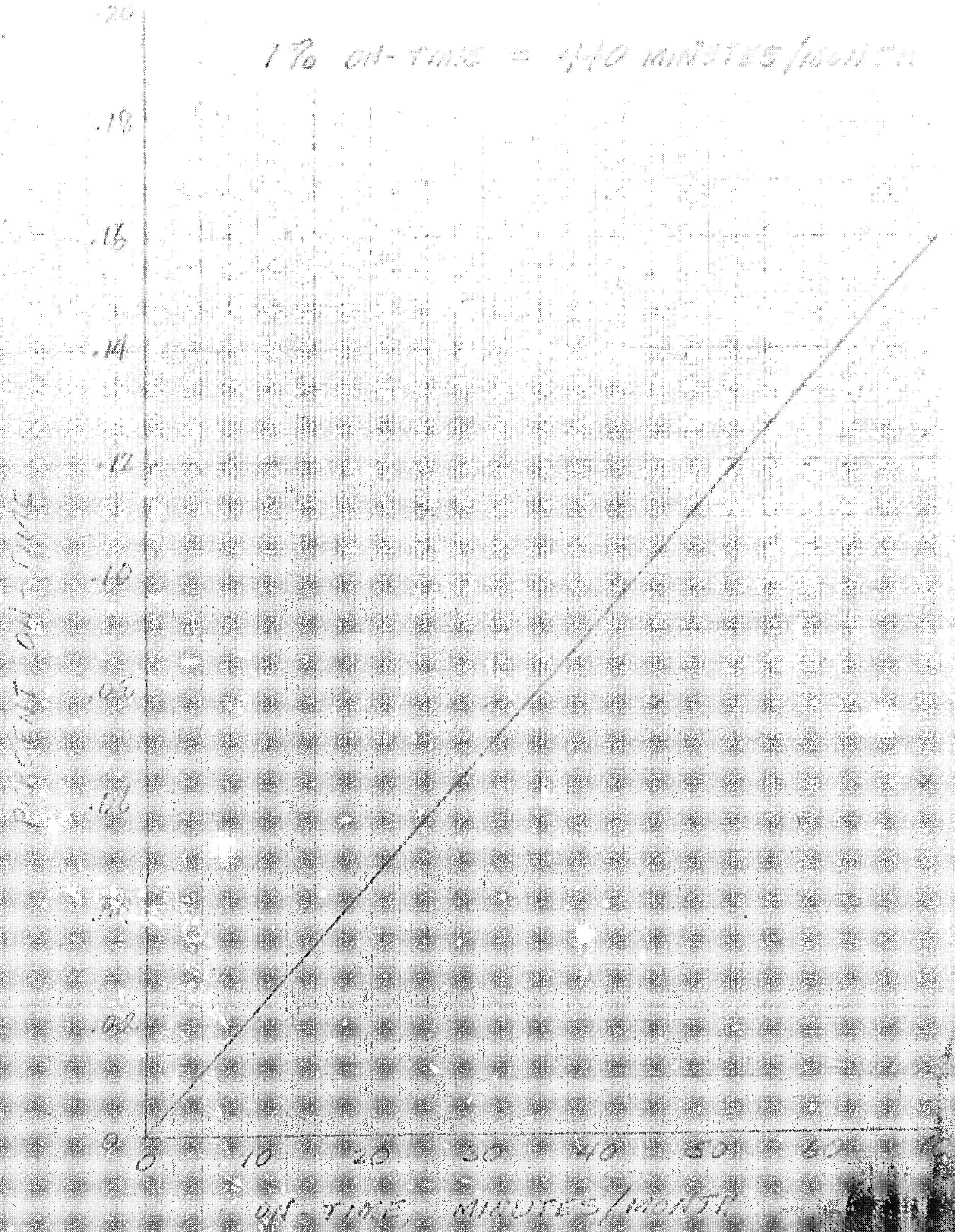


FIGURE 5. PERCENT ON-TIME

Analysis

The average probability that a given point on the earth will be within the horizon coverage circle of a given satellite system at any arbitrary instant is

$$P = T/1440 = \bar{t}\,N/1440$$

where T is the average number of minutes per day that the given point is visible from the satellite orbit and 1440 is the number of minutes in a day. A way to compute T is to compute the average number of orbits per day, N, during which the given point appears above the horizon and to multiply it by the average LOS visibility time per contact, $\bar{t}$. For a non-repeating ground track orbit the results are independent of target longitude. A way to compute N for a given orbit and a given target latitude is to graphically determine (as on an orbit planning board or by ground trace plots) the number of degrees of longitude at the equator, $\Delta\theta$, over which contact will occur within the next orbital revolution then compute

$$N = (\Delta\theta/360)\cdot(\text{Orbits/Day})$$

Now consider a 275 NM circular orbit. Neglecting the slight effect of earth rotation (which depends on orbital inclination and target latitude) the tracking time and elevation angle to this orbit are plotted in Figure 6 as a function of the cross track distance to the target signal of interest (SOI) at the point of closest approach, $D_{Min.}$ The horizon field of view has a radius of 1350 NM. The LOS visibility time for an overhead pass is 11.6 minutes. The average visibility time for all passes within a cross-track distance, D, is

$$\bar{t} = \frac{11.6}{2}\left\{\left[1 - \left(\frac{D}{1350}\right)^2\right]^{\frac{1}{2}} + \left[\frac{1350}{D}\sin^{-1}\frac{D}{1350}\right]\right\} \text{ minutes}$$

which has a value of 9.1 minutes for all passes above the 0° horizon (D = 1350). The average time above the 0° horizon for all passes which provide a minimum tracking time, m, is as follows:

m =	0	1	3	5	8	10	11 min.
D =	1350	1345	1300	1200	980	700	450 NM
$\bar{t}$ =	9.1	9.1	9.4	9.8	10.5	11.1	11.44 min.

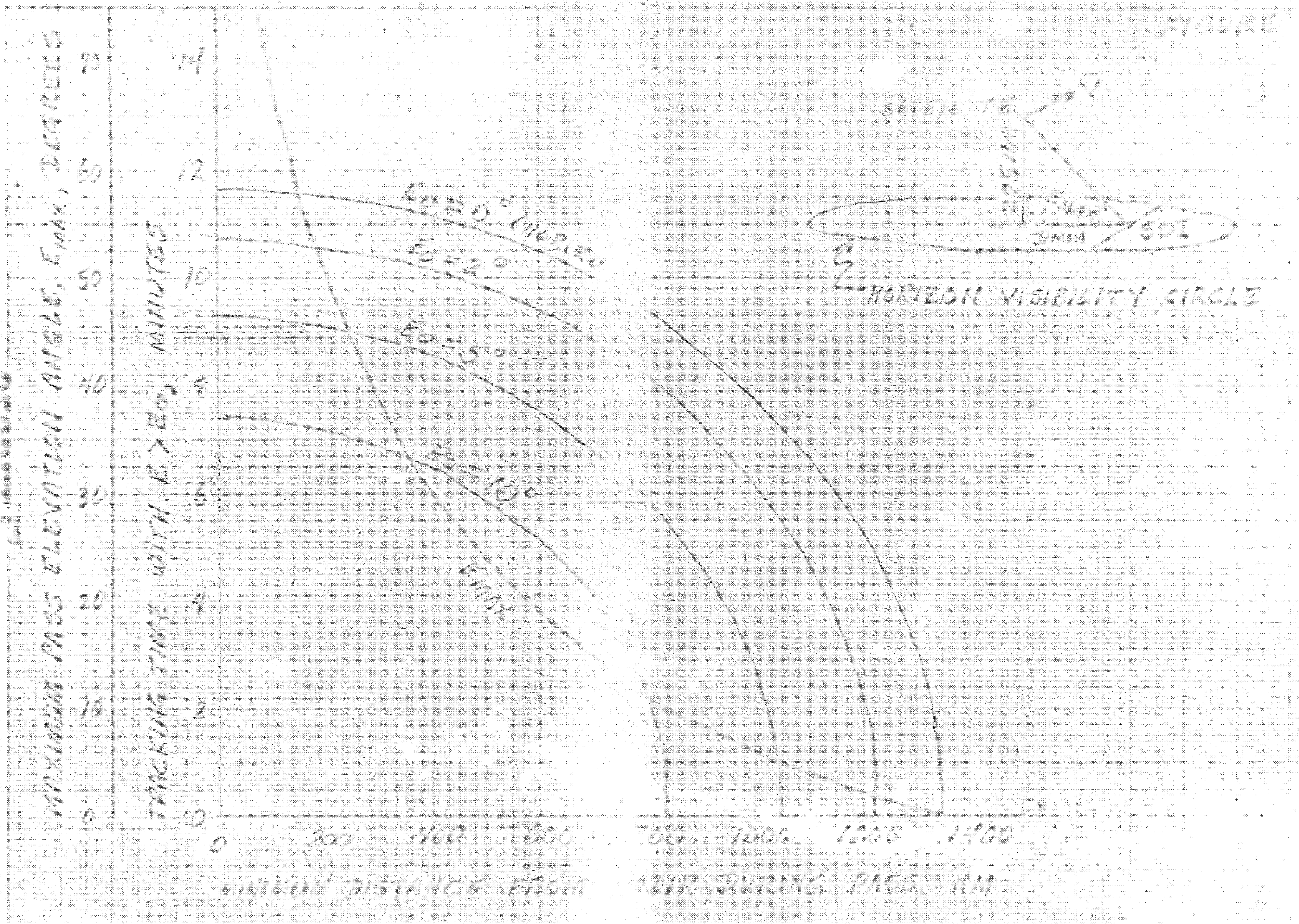


FIGURE 6. CONTACT PARAMETE[RS] FROM 275 KM ORBIT

As an example of the use of these results, let us compute the probability of obtaining a complete 5 minute intercept on an SOI which turns on at an arbitrary time and stays on for 5 minutes from an orbit which provides only 4.9 passes per day (on average) of at least 5 minutes visibility from the SOI location. The result is

$$P = (\bar{T} - m)\, N/1440 = (9.8 - 5)\cdot(4.9)/1440 = .0163$$

This point is the origin of the "contact time $\geq$ 5 minutes" curve in Figure 2. On the same plot, the number of passes per day above the 0° horizon is 5.5, and the probability of at least fleeting contact (time $\geq$ 0) with some part of an SOI which is on for 5 minutes is

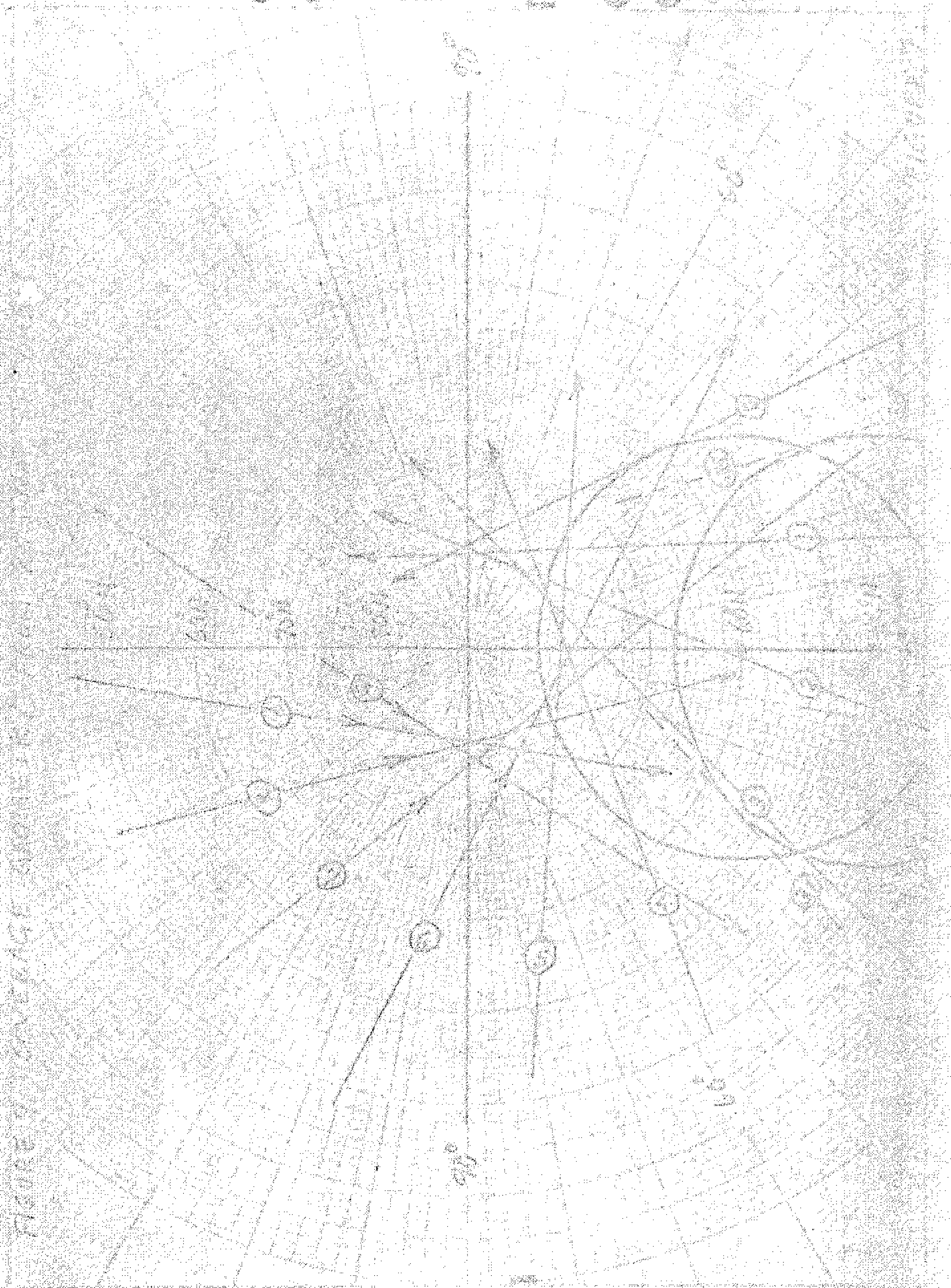
$$P = (\bar{T} + 5)\, N/1440 = (9.3 + 5)\,(5.5)/1440 = .054$$

as shown in Figures 1 and 2 for the SOI at 45°N which is active for 5 minutes.

The number of passes per day above the SOI 0° horizon is shown in Figure 4 as a function of SOI latitude. As indicated above, this number of passes decreases if we require that each pass provide more than some specified number of minutes of SOI visibility. The number would also decrease (as would the average visibility times) if we used some elevation angle greater than zero degrees to define the useful horizon (such as the 5° curve in Figure 6 for example). The geometrical effects which dictated the shape of the curves of Figure 4 are illustrated in Figure 7. The horizon coverage circles for SOI latitudes of 45°N and 60°N are shown with sample ground traces for a 275 NM, 80° orbit. Orbit zero is arbitrarily defined as that pass which provides fleeting contact with the SOI at 60°N. As indicated, if this pass occurred 0.35 orbital periods later, then it would provide fleeting contact with the SOI at 45°N. It takes 2.75 orbital periods for earth rotation to move the SOI at 45°N through the region of contact (pass 0.35 to pass 3.1). This occurs again between passes 6 and 10. Hence, there are 5.5 passes per day above the 45°N SOI horizon.

Passes 4 through 6 are too far north to be seen from the 45°N SOI position, but the coverage circle for the 60°N SOI extends to 82°N and thus provides an unbroken series of 9.7 passes as the orbit plane walks across the SOI position. An orbit with a 60° inclination obtains this type of coverage against an SOI at 45°N. It is the above behavior that causes the curves of Figure 4 to break at the latitudes shown. Since latitudes above 82°N are below the 275 NM horizon from 60°N, the number of visible passes per day from the 60° orbit goes to zero at an SOI latitude of 82°N.

With information of the above type, the computation of Figures 1 and 2 is straightforward. For SOI on-times less than the pass-to-pass dead time (orbital period minus visibility time) the probability of contact is a simple ratio of average contact time per day to the length of the day. For on-times which exceed the



pass-to-pass dead time the probability is $1 - P_o$ where P_o is the probability of the on-time occurring during those passes which are on the far side of the pole from the SOI location. For on-times which exceed the time span of these non-contact passes (less than 1/2 day) the probability of contact is unity (as shown in Figures 1 and 2).

Figures 1 and 2 give the probability of contact per SOI turn-on. Figure 3 gives the cumulative probability of contact as a function of the probability per turn-on and the number of turn-ons. As long as the SOI does not have a preferred time of day for turn-on and as long as the turn-ons are a number of days apart, then the probabilities are independent from one turn-on to the next. Given the single turn-on probability, P_1, the cumulative probability for N turn-ons is then

$$P = 1 - (1 - P_1)^N \quad \text{or} \quad N = \frac{\log (1 - P)}{\log (1 - P_1)}$$

The second form of this expression allows the required N to be computed for any desired P (as for 95% probability).

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