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BASELINE DOCUMENT FOR TSPOOND

I. Introduction and General Philosophy

The purpose of the computer program TSPOOND (Statistical Program of On-Board Decisions) is to simulate the actual conditions of target visibility and activity, crew voting and consequent main optics selections for the MOL/DORIAN missions. The program will accomplish the simulation using a Monte Carlo technique that will take random samples of certain variables such as target visibility, activity, scope dwell times, crew recognition and false alarm. The distribution functions for the above random variables will be input so that studies may be performed to determine the effect that changes in the distributions have upon mission results. A primary output of TSPOOND will be a comparison of the manned vs. unmanned results.

The primary input to the program will be the BIT (Binary Interface Tape) produced by TWONDER. In addition, various software control parameters and output commands will be processed via card input.

The general structure of the program will be designed so that the user can specify both (1) the important input parameters that describe crew performance and voting strategies, and (2) the desired output quantities including those from a typical sample and/or statistical summaries for a large number of samples.

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Notation and Definition of Variables

Non-random Variables

Definition

P_V	Probability that a target is visible, target specific
$T^{(i)}$	Group decision time for group i (in machine time)
Δt	Recommended dwell time designated by mission planning
δt_s	Acquisition scope slew and settle time from previous target
P_α	Probability that a target is active, target specific
P_R	Probability that a crew member recognizes activity when it is present, scope specific
P_F	Probability of false alarm (i.e. probability that a crew member reports a target active when it is inactive), scope specific
ρ	Intra-group visibility correlation coefficient

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The following set of random variables will be sampled from in the program:

<u>Random Variable</u>	<u>Definition</u>
V	Target visibility, $P_r \{V=1\} = P_V$ $P_r \{V=0\} = 1 - P_V$
δt	Scope viewing time for a target $\delta t = \delta t_s + \delta t_r$
α	Target activity, $P_r \{\alpha=1\} = P_\alpha$ $P_r \{\alpha=0\} = 1 - P_\alpha$
ξ	Recognition, $P_r \{\xi=1\} = P_R$ $P_r \{\xi=0\} = 1 - P_R$
η	False alarm, $P_r \{\eta=0\} = P_F$ $P_r \{\eta=1\} = 1 - P_F$

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Monte Carlo Cycle Length

The Monte Carlo cycle length for TSPOOND will be one rev, i.e., all the random samples will be processed for one rev before beginning on the next. One advantage of such a short cycle length is the possibility of looking at the results from one rev before making a decision to complete the entire run. It should be emphasized that this choice for cycle length does not preclude getting statistics for quantities over several revs.

Random Number Generators

Two types of random number generators will be used by TSPOOND to generate the (pseudo) random numbers needed to compute the random quantities involved in the Monte Carlo sampling. The first random number generator (RNG) generates random numbers between 0 and 1 with a uniform distribution. The second RNG generates random numbers from an input cumulative distribution function. This second RNG is needed to compute the random dwell times for the acquisition scopes.

Many of the variables in the program are so-called "zero-one" random variables that assume the value 0 with some probability α_1 and 1 with probability $1 - \alpha_1$. If Q is such a variable, its value for each sample is determined by generating a number r from the uniform RNG on $[0, 1]$ and comparing it with α_1 . If $r < \alpha_1$, Q is set to 0. If $r > \alpha_1$, Q is set to 1. That Q has a probability α_1 of being equal to 0 follows from the relation

$$P_r(r < \alpha_1) = P_r(Q=0) = \alpha_1$$

since r is uniformly distributed on $[0, 1]$.

Simulation of Acquisition Scope Dwell Times

The amount of time that a crew member spends viewing a target is a variable that depends on whether or not the target is visible, and, if visible, on whether or not the crew member notices activity. TWONDER uses a recommended dwell time for each target which will be transmitted to TSPOOND on the BIT. This recommended dwell time will then be used as a basis

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for determining a distribution function for the sample dwell time. Thus if δt_r represents the actual dwell time and δt^* represents the recommended dwell time, the distribution function for the random variable δt_r will be represented by some function $f(\delta t_r)$. $\times f(\delta t^*)$

However, the amount of time required to recognize a cloud is less than that required to view a target and search for activity, so that TSPOOND will use a different distribution function for δt_r if the target is not visible.

Since very little detailed information is available on the shape of f , TSPOOND will allow the form of the distribution functions to be specified by input so that they may be altered for parametric studies or to better simulate crew actions.

How?

Crew Voting Logic

One purpose of the Monte Carlo program is to study the effect of different voting logics on the mission results. To that end, TSPOOND will maintain for each group a voting table V with four columns and up to eight rows. The four columns will correspond to the votes of active, inactive, reject, and override, respectively. TSPOOND will either fill-in the tables from weights on the BIT or will read them from input cards. The program will allow the user to specify what numbers he wishes to use for unobserved targets and whether or not any targets are classed as mandatories. A mandatory target (no more than one per group) will automatically be chosen for photography and no voting will be simulated for its group. One logical way of choosing the weights for the voting is to let the active and inactive columns be the expected weights of the targets given that they are in the reported state, the reject column be all zeros, and neglect the override column. This method would maximize the expected score. However, it should be emphasized that the program will be used to study different types of voting and will be designed accordingly.

Is this a capability?

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Probability of Visibility

One of the major factors influencing the decision of what target in a group to photograph is the weather conditions in the area of the group. The weather, or more strictly, the presence or absence of cloud cover, affects the vote that is made for each target by the crew members. The program requires for each target a probability of visibility, denoted P_v , which reflects the percentage of times (on the average) that the crew member will see the target out of the total number of times that he views it.

Since cloud cover is clearly a regional phenomenon, it is desirable to reflect this in the computation of P_v for each target.

Two types of correlation may be considered, namely, inter-group correlations and intra-group correlations. Inter-group correlations take into account that the weather of two groups close to each other is correlated. Intra-group correlations take into account that the weather of two targets in the same group (and hence geographically quite close) is correlated.

How do we correlate if all targets are different P.v.?

A detailed account of the methods of introducing the correlations and the conditional probabilities that result are included in Appendices A and B.

Conflict Analysis

Although the determination of groups guarantees that the primary of group $i + 1$ doesn't conflict with any target in group i , and that the primary of group i doesn't conflict with any target in group $i + 1$, it is possible for an alternate in group i to conflict with an alternate in group $i + 1$. TWONDER will compute the conflict relationships for each group and pass the information to TSPOOND on the BIT. The simulation in TSPOOND will then remove from consideration any target in group $i + 1$ that conflicts with the target chosen for photography in group i .

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III. Inputs

The primary input to TSPOOND is the BIT (Binary Interface Tape) generated by TWONDER. In addition, various software control parameters and the specification of output parameters will be input on cards.

The BIT from TWONDER will contain the following information:

1. The path selection for the main optics and two acquisition telescopes, including the recommended dwell time and on and off times for the scopes. The targets will be arranged by group.
2. The active and inactive weights of the targets
$$w_A^{(i)}, w_I^{(i)}$$
3. The probabilities of activity, visibility.
4. The target card parameters (priorities, visual intelligence only flags, etc.).
5. The group decision time for each group.
6. Identification of run and rev span.

The second class of input data includes software control parameters and output specifications. The amount of data of this type is small and will be input on cards; the data will also be included in the output to clearly identify the run.

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IV. Output

The output from TSPOOND will consist of two general types of results for a specified rev span: (1) a typical sample giving detailed information about the actual status of the variables, voting, and decisions made, and (2) statistical summary data derived from all the samples including the average number of interdictions, average scores for manned and unmanned modes, and the average number of unobserved alternates.

All of the output will be written on the output tape (SPOUT) in BCD and may be printed if desired. The rev spans and requested output parameters will be specified as input to TSPOOND, but will also be written at the beginning of the SPOUT to clearly identify its contents.

In order to ascertain what a typical mission looks like, TSPOOND will output, on option, a detailed account of specified rev spans. Included in this output will be:

1. The actual and reported states of every scheduled target
2. The target chosen for photography in each group.
3. The scope on and off times.
4. The main optics on and off times for each target.
5. The group decision time, group number, rev number, scope number.
6. The mid-time stereo angle, slant range, obliquity and photo mode.
7. The active and inactive priorities.
8. The VIT and primary flags.

In order to determine what the most common results and variations will be, TSPOOND will output statistical summary data for a large number of samples. Means, variances, cumulative distribution function plots, maximum and minimum values and percentile information may be obtained for the important output quantities. The number of samples to be taken, a specification of what statistical information is desired, and the rev span of interest will be specified as input to TSPOOND and written on the SPOUT for reference.

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Some of the quantities presently of interest are:

1. The average number of primaries selected for main optics photography.
2. The average number of alternates selected and why they were selected.
3. The average manned and unmanned score.
4. The average number of primaries not selected and why.
5. The average number of adjacent alternates (if no conflict analysis is conducted)
6. The average number of unobserved alternates.
7. The number of mandatories specified.
8. The average number of overrides.

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APPENDIX A

Intra-group Correlation Model

New random variable: $G = \begin{cases} 1 & \text{if group is in a "clear" area} \\ 0 & \text{if group in a "cloudy" area} \end{cases}$

let $P_G = P_r(G = 1)$

$V_i = \begin{cases} 1 & \text{if } T_i \text{ is visible} \\ 0 & \text{if not} \end{cases}$

let $P_{V_i} = P_r(V_i = 1)$ (1)

The V_i depend on G in the following way:

$$P_r(V_i = 1 \mid G = 1) = r_1 \quad (2)$$

$$P_r(V_i = 0 \mid G = 1) = 1 - r_1$$

$$P_r(V_i = 1 \mid G = 0) = r_2 \quad (3)$$

$$P_r(V_i = 0 \mid G = 0) = 1 - r_2$$

The non-conditional probabilities of visibility are:

$$P_r(V_i = 1) = (P_G)(P_r(V_i = 1 \mid G = 1)) + (1 - P_G)(P_r(V_i = 1 \mid G = 0))$$

$$= P_G r_1 + (1 - P_G) r_2 = P_r(V_j = 1) \quad (4)$$

$$P_r(V_i = 0) = P_G (1 - r_1) + (1 - P_G)(1 - r_2) \quad (5)$$

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A posteriori:

$$\begin{aligned} P_r(G = 1 \mid V_i = 1) &= \frac{P_r(G = 1 \text{ and } V_i = 1)}{P_r(V_i = 1)} \\ &= \frac{P_G r_1}{P_G r_1 + (1 - P_G) r_2} \end{aligned} \quad (6)$$

$$\begin{aligned} P_r(G = 1 \mid V_i = 0) &= \frac{P_r(G = 1 \text{ and } V_i = 0)}{P_r(V_i = 0)} = \frac{P_G (1 - r_2)}{P_G (1 - r_1) + (1 - P_G) (1 - r_2)} \end{aligned} \quad (7)$$

$$\begin{aligned} P_r(G = 0 \mid V_i = 1) &= \frac{P_r(G = 0 \text{ and } V_i = 1)}{P_r(V_i = 1)} = \frac{(1 - P_G) r_2}{P_G r_1 + (1 - P_G) r_2} \end{aligned} \quad (8)$$

$$\begin{aligned} P_r(G = 0 \mid V_i = 0) &= \frac{P_r(G = 0 \text{ and } V_i = 0)}{P_r(V_i = 0)} = \frac{(1 - P_G) (1 - r_2)}{P_G (1 - r_1) + (1 - P_G) (1 - r_2)} \end{aligned} \quad (9)$$

Joint:

$$P_r(V_i = 1 \text{ and } V_j = 1) = P_G r_1^2 + (1 - P_G) r_2^2 \quad (10)$$

$$P_r(V_i = 1 \text{ and } V_j = 0) = P_G r_1 (1 - r_1) + (1 - P_G) (r_2 (1 - r_2)) \quad (11)$$

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$$= P_r(V_i = 0 \text{ and } V_j = 1)$$

$$P_r(V_i = 0 \text{ and } V_j = 0) = P_G(1 - r_1)^2 + (1 - P_G)(1 - r_2)^2 \quad (12)$$

Conditional:

$$\begin{aligned} P_r(V_i = 1 \mid V_j = 1) &= \frac{P_r(V_i = 1 \text{ and } V_j = 1)}{P_r(V_j = 1)} \\ &= \frac{P_G r_1^2 + (1 - P_G) r_2^2}{P_G r_1 + (1 - P_G) r_2} \end{aligned} \quad (13)$$

$$\begin{aligned} P_r(V_i = 0 \mid V_j = 1) &= \frac{P_r(V_i = 0 \text{ and } V_j = 1)}{P_r(V_j = 1)} \\ &= \frac{P_G r_1(1 - r_1) + (1 - P_G) r_2(1 - r_2)}{P_G r_1 + (1 - P_G) r_2} \end{aligned} \quad (14)$$

$$\begin{aligned} P_r(V_i = 0 \mid V_j = 0) &= \frac{P_r(V_i = 0 \text{ and } V_j = 0)}{P_r(V_j = 0)} \\ &= \frac{P_G(1 - r_1)^2 + (1 - P_G)(1 - r_2)^2}{P_G(1 - r_1) + (1 - P_G)(1 - r_2)} \end{aligned} \quad (15)$$

$$P_r(V_i = 1 \mid V_j = 0) = \frac{P_G r_1(1 - r_1) + (1 - P_G) r_2(1 - r_2)}{P_G(1 - r_1) + (1 - P_G)(1 - r_2)} \quad (16)$$

The generation of the visibilities V_i for each group is then accomplished by the general method described in Reference [1].

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APPENDIX B

Inter-group Correlation Model

The correlation coefficient between the probability of visibility for group i, $P_V^{(i)}$, and the probability of visibility for group i + 1, $P_V^{(i+1)}$, will be determined by the time between their respective group decision times. A quantity t_{max} will be input and represents the time at which the inter-group correlation falls to 0. A linear decay is assumed so that

$$\rho(t) = \begin{cases} 1 - \frac{t}{t_{max}} & \text{if } t < t_{max} \\ 0 & \text{if } t \geq t_{max} \end{cases} \quad (1)$$

where t represents the time interval between the two group decision times.

One constraint must be applied to (1) that is inherent to the problem,

$$\rho(t) = \min \left(\frac{P_V^{(i)} (1 - P_V^{(i+1)})}{P_V^{(i+1)} (1 - P_V^{(i)})}, \frac{P_V^{(i+1)} (1 - P_V^{(i)})}{P_V^{(i)} (1 - P_V^{(i+1)})} \right)$$

Thus, the correct definition of $\rho(t)$ is

$$\rho(t) = \begin{cases} \min \left(1 - \frac{t}{t_{max}}, \frac{P_V^{(i)} (1 - P_V^{(i+1)})}{P_V^{(i+1)} (1 - P_V^{(i)})}, \frac{P_V^{(i+1)} (1 - P_V^{(i)})}{P_V^{(i)} (1 - P_V^{(i+1)})} \right) & \text{if } t < t_{max} \\ 0 & \text{if } t \geq t_{max} \end{cases}$$

Once this correlation coefficient has been computed, the group visibilities will be computed by the general method described in Reference (2).

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REFERENCES *at the time*

1. P. L. McEntire, "A Method of Generating a Correlated Set of N Zero-One Random Variables with a Specified Mean," TRW Systems, Sunnyvale.
2. P. L. McEntire, "A Method of Generating Two Zero-One Random Variables with Specified Means and Correlation Coefficient," TRW Systems, Sunnyvale.

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INTEROFFICE CORRESPONDENCE

TO: T. A. Magness

CC:

DATE: 21 August 1968

SUBJECT: A Method of Generating Two Zero-One Random
Variables with Specified Means and Correlation
Coefficient

FROM: P. L. McEntire
BLDG. MAIL STA.
Sunnyvale Office

The purpose of this memorandum is to describe a method for
generating two random variables, x_1 and x_2 , so that:

$$\begin{aligned} \text{(A)} \quad P_r\{x_1 = 1\} &= \mu_1 & [P_r\{x_1 = 0\} &= 1 - \mu_1] \\ P_r\{x_2 = 1\} &= \mu_2 & [P_r\{x_2 = 0\} &= 1 - \mu_2] \\ \text{(B)} \quad \rho(x_1, x_2) &= \bar{\rho} \end{aligned}$$

The values of μ_1 , μ_2 , and $\bar{\rho}$ are numbers in $[0, 1]$ subject only to the
constraint

$$\bar{\rho} \leq \min \left\{ \sqrt{\frac{\mu_1(1-\mu_2)}{\mu_2(1-\mu_1)}}, \sqrt{\frac{\mu_2(1-\mu_1)}{\mu_1(1-\mu_2)}} \right\} \quad (1)$$

The constraint (1) says essentially that if μ_1 and μ_2 are very far
apart, then the correlation cannot be very high.

The value of x_1 will first be selected and then the value of x_2 will
be selected conditionally so that

$$\begin{aligned} P_r\{x_2 = 1 \mid x_1 = 1\} &= P_1 & [P_r\{x_2 = 0 \mid x_1 = 1\} &= 1 - P_1] \\ P_r\{x_2 = 1 \mid x_1 = 0\} &= P_2 & [P_r\{x_2 = 0 \mid x_1 = 0\} &= 1 - P_2] \end{aligned}$$

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From condition (A) and the definition of expectation

$$E(x_2) = \mu_2 = \mu_1 P_1 + (1 - \mu_1) P_2 \quad (2)$$

Also the variances of x_1 and x_2 are given by

$$\sigma_{x_1}^2 = \mu_1(1 - \mu_1)$$

$$\sigma_{x_2}^2 = \mu_2(1 - \mu_2)$$

and

$$E(x_1 x_2) = \mu_1 P_1$$

Hence the correlation coefficient between x_1 and x_2 is

$$\begin{aligned} \rho(x_1, x_2) &= \frac{\text{COV}(x_1, x_2)}{\sigma_{x_1} \sigma_{x_2}} = \frac{E(x_1 x_2) - E(x_1) E(x_2)}{\sigma_{x_1} \sigma_{x_2}} \\ &= \frac{\mu_1 P_1 - \mu_1 \mu_2}{\sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)}} \end{aligned} \quad (3)$$

And so from condition (B),

$$\bar{\rho} = \frac{\mu_1 P_1 - \mu_1 \mu_2}{\sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)}} \quad (4)$$

Thus the problem has reduced to the simultaneous solution of (2) and (4) for P_1 and P_2 . Solving (4) for P_1 yields

$$P_1 = \frac{\bar{\rho}}{\mu_1} \sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)} + \mu_2 \quad (5)$$

Substituting P_1 from (5) into (4),

$$P_2 = \mu_2 - \frac{\bar{\rho}}{1 - \mu_1} \sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)} \quad (6)$$

The solutions (5) and (6), however, must fall between 0 and 1. Subjecting (5) to this constraint yields

$$\frac{\bar{\rho}}{\mu_1} \sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)} + \mu_2 < 1 \quad (7)$$

$$\frac{\bar{\rho}}{\mu_1} \sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)} < 1 - \mu_2$$

$$\bar{\rho} < \frac{\mu_1(1 - \mu_2)}{\sqrt{\mu_1(1 - \mu_1) \mu_2(1 - \mu_2)}}$$

and finally

$$\bar{\rho} < \sqrt{\frac{\mu_1(1 - \mu_2)}{\mu_2(1 - \mu_1)}} \quad (8)$$

which is one-half of the constraint mentioned above in (1).

A similar computation from (6) results in

$$\bar{\rho} < \sqrt{\frac{\mu_2(1 - \mu_1)}{\mu_1(1 - \mu_2)}} \quad (9)$$

Thus (8) and (9) together yield (1).


P. L. McEntire

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INTEROFFICE CORRESPONDENCE

TO: T. A. Magness

CC:

DATE: 19 August 1968

SUBJECT: A Method of Generating a Correlated Set of N
Zero-One Random Variables with a Specified Mean

FROM: P. L. McEntire /
BLDG. MAIL STA. EX:
Sunnyvale Office

This memorandum describes a method of generating N random variables x_1, \dots, x_N such that:

$$(A) \quad P_r\{x_i = 1\} = \bar{x}$$

$$P_r\{x_i = 0\} = 1 - \bar{x} \quad \text{for } i=1, 2, \dots, N$$

$$(B) \quad \rho(x_i, x_j) = \bar{\rho} \quad \text{for } i \neq j$$

The values of \bar{x} and $\bar{\rho}$ are arbitrary but fixed.

First an auxiliary zero-one random variable C is introduced with $P_r\{C = 1\} = P_C$ and $P_r\{C = 0\} = 1 - P_C$. Then the distribution functions for the x_i are determined conditionally as functions of C so that:

$$P_r\{x_i = 1 \mid C = 1\} = P_1 \quad (P_r\{x_i = 0 \mid C = 1\} = 1 - P_1)$$

$$P_r\{x_i = 0 \mid C = 0\} = P_1 \quad (P_r\{x_i = 1 \mid C = 0\} = 1 - P_1)$$

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The method consists of determining P_C and P_1 so that conditions (A) and (B) are satisfied. Condition (A) implies that

$$E(x_i) = P_C P_1 + (1 - P_C)(1 - P_1) = \bar{x} \quad (1)$$

Now the covariance of x_i and x_j is given by

$$\text{COV}(x_i, x_j) = E(x_i x_j) - E(x_i) E(x_j)$$

where

$$E(x_i x_j) = P_C P_1^2 + (1 - P_C)(1 - P_1)^2$$

and

$$E(x_i) = E(x_j) = P_C P_1 + (1 - P_C)(1 - P_1)$$

So carrying out the algebra,

$$\text{COV}(x_i, x_j) = P_C(1 - P_C)(2P_1 - 1)^2 \quad (2)$$

Finally, the correlation coefficient between x_i and x_j is given by

$$\rho(x_i, x_j) = \frac{\text{COV}(x_i, x_j)}{\sigma_{x_i} \sigma_{x_j}} \quad (3)$$

$$\sigma_{x_i}^2 = E(x_i^2) - [E(x_i)]^2 = E(x_i)[1 - E(x_i)]$$

So (3) becomes

$$\rho(x_i, x_j) = \frac{P_C(1 - P_C)(2P_1 - 1)^2}{E(x_i)[1 - E(x_i)]} \quad (4)$$

If condition (B) is now applied to (4),

$$\bar{\rho} = \frac{P_C (1-P_C) (2P_1-1)^2}{E(x_i) [1-E(x_i)]} \quad (5)$$

Rewriting (1),

$$\bar{x} = P_C P_1 + (1 - P_C)(1 - P_1) = E(x_i) \quad (6)$$

Thus the problem reduces to the solution of two simultaneous equations (5) and (6), for P_C and P_1 .

Substituting \bar{x} for $E(x_i)$ in (5) yields

$$\bar{\rho} = \frac{P_C (1-P_C) (2P_1-1)^2}{\bar{x}(1-\bar{x})} \quad (7)$$

$$\text{Let } K = \bar{\rho} \bar{x}(1 - \bar{x}) \quad (8)$$

Then (7) becomes

$$K = (P_C - P_C^2) (2P_1 - 1)^2 \quad (9)$$

Solving (6) for P_1 ,

$$P_1 = \frac{\bar{x} + P_C - 1}{2P_C - 1} \quad (10)$$

Back to (9),

$$K = (P_C - P_C^2) \left(\frac{2\bar{x} + 2P_C - 2}{2P_C - 1} - 1 \right)^2 \quad (11)$$

$$K = (P_C - P_C^2) \left(\frac{2\bar{x} - 1}{2P_C - 1} \right)^2 \quad (12)$$

Now let

$$K' = \frac{K}{(2\bar{x} - 1)^2} \quad (13)$$

Equation (12) becomes

$$K'(2P_C - 1)^2 = P_C - P_C^2$$

$$K'(4P_C^2 - 4P_C + 1) = P_C - P_C^2$$

Whence finally

$$P_C^2 - P_C + \frac{K'}{4K' + 1} = 0 \quad (14)$$

Either root of (14) may be chosen for P_C , so, without loss of generality let

$$P_C = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{4K'}{4K' + 1}}$$

$$P_C = \frac{1}{2} + \frac{1}{2} \sqrt{\frac{1}{4K' + 1}} \quad (15)$$

Substituting P_C into (10) yields

$$P_1 = \frac{1}{2} + (\bar{x} - \frac{1}{2}) \sqrt{4K' + 1} \quad (16)$$

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The values of P_C and P_1 from (15) and (16) yield the N random variables x_i satisfying conditions (A) and (B). The method is useful for computational purposes because the generation of the x_i can be accomplished by a succession of samples from a simple Bernoulli random number generator.

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