~~TOP SECRET RUFF~~

GOVERNMENT

SYSTEMS

FINAL REPORT Project [redacted]

October 1966

A PLAN FOR EVALUATING THE ACCURACY
OF A MULTIPLE ORBIT ADJUSTMENT

Prepared for

U.S. Army Engineers Geodesy,
Intelligence, Mapping Research
and Development Agency
Fort Belvoir, Virginia

Declassified and Released by the N R O

In Accordance with E. O. 12958

on NOV 26 1997

Itek

NOTICE — ~~THIS DOCUMENT / MATERIAL CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. ITS TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.~~

Itek Corporation

DATA ANALYSIS CENTER

610 Franklin Street, Alexandria, Virginia

~~TOP SECRET RUFF~~

HANDLE VIA ~~TALENT KEYHOLE~~ CONTROL SYSTEM

ABSTRACT

The purpose of this report is to establish a procedure for evaluating the accuracy of position determinations accomplished by satellite photogrammetry. While this test plan is directed specifically toward the multiple orbit adjustment of the existing DAFF photography it is sufficiently general to be applied to any photogrammetric adjustment employing orbital constraints.

The method employed consists of removing systematic errors so that accuracy and precision become nearly equivalent and then performing a complete error propogation in the photogrammetric intersection of points whose positions are known. The checking for systematic errors is accomplished by separating the residuals from the orbit adjustment into subsets based on similarity of observing conditions and comparing the mean and standard deviations of each subset with the same statistics associated with the whole set of residuals. Statistical tests are then employed to determine whether or not the deviation of this subset from the whole could be expected to result from chance. If it cannot, the physical characteristics of the subset provides a clue to the type of systematic error encountered. Methods for the removal of many of these systematic errors are given.

In intersecting the ground points whose geodetic positions are known, observational errors due to the photo coordinates, orientation angles, and orbital positions are carried through the statistical analysis. Furthermore, the covariance among the several exposure station positions is employed so that a true estimate of the expected error is

[redacted]

obtained. Statistical statements can then be made about the distance of the true points from their computed counterparts, and since true position is known the validity of the statistical analysis is readily checked. Since these check points are well distributed over the land surface of the earth, the accuracy of computed terrain point positions can be obtained as a function of latitude and longitude.





HANDLE VIA
TALENT-KEYHOLE
CONTROL SYSTEM ONLY

# 1. INTRODUCTION

The purpose of this report is to set forth a procedure to be used to appraise the accuracy of a simultaneous adjustment of data from several satellite missions. Although this test plan is aimed at evaluating two specific programs, the [redacted] RECAP Program and the [redacted] TRACE-D Program, and the results of adjustments by these programs of existing DAFF photography; sufficient generality is maintained so that the same test procedure can be applied in the evaluation of similar results from future adjustments. The technique employed consists of (1) The removal of systematic errors, (2) Use of the adjusted orbital camera stations to compute the positions of known geodetic control points and the accompanying propogation of random errors in the computed positions, and finally (3) Evaluation of the predicted magnitude of the errors in comparison with the known difference between true and computed positions. At the present time the areas in which geodetic control points are available may not provide a geometric distribution that will permit a conclusive test to be performed, but as more and better control becomes available in strategic areas greater confidence can be placed in the results of such a test.

Tests which have been conducted, or are in progress at the Army Map Service to evaluate the accuracy of each individual mission adjustment have been consulted in the preparation of this report. The procedures employed in these tests and the standardized technique that has evolved in the course of solving some of the operational problems encountered have been very helpful. Furthermore, much of the data to be used in the test herein described has already been gathered for use in evaluating the adjustment of individual missions.

The need for a plan for testing the accuracy of results obtained from the simultaneous adjustment of all DAFF missions was first recognized by [redacted] of G.I.M.R.A.D.A. It was through his efforts and those of several individuals at O.C.E. and the Army Map Service that such a study was initiated and a contract subsequently awarded to the Itek Corporation to design this plan of test. This effort has been made possible through the cooperation of many people at the Army Map Service; principally Messers [redacted]. Itek personnel who have contributed to the development include Messers [redacted] while [redacted] and [redacted] provided assistance in areas of photogrammetry and celestial mechanics that could be considered unclassified. The final compilation of material and writing of the text was done by [redacted].





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY

## 2. ACCURACY AND PRECISION

When statistical statements are made about the quality of the results obtained from an adjustment employing real data, it is usually precision that is being discussed. Precision is the estimate of the errors remaining in the final results due to random errors in the original observations. By adding to the precision estimate the systematic error, which is probably unknown, an estimate of the accuracy of the final results can be made. Hence, if all systematic errors can be removed or reduced below a significant level. the standard error propogation formulas employed by staticians will yield estimates of accuracy -- accuracy and precision being equivalent under these circumstances.

### 2.1. Detection of Systematic Errors

The purpose of the Least Squares adjustment process is to minimize the sum of the squares of redisuals in the observed variables. Although the use of a least squares technique does not require that any severe restrictions be placed on the distribution of these residuals, it will be found convenient to assume that they are normally distributed. If such an assumption is found to be justified, the results obtained from a weighted least squares adjustment will be the same as would result from a maximum likelihood adjustment, and the more convenient set of statistical tests based on the assumption of a normal distribution of errors can be applied. However, an even more basic assumption on which all statistical tests are based is that the errors in the observed data are random, i.e., that all systematic errors have been removed. It will be necessary to perform extensive tests to ascertain whether or not all systematic errors have been reduced below a negligible level, and to evaluate the magnitude of any residual systematic error so that it can be compensated for.

Since the residuals obtained from the RECAP Program are weighted, that is each residual has been divided by the standard deviation associated with its observation type, the observation residuals form a population that is believed to be normally distributed with a mean of zero and a unit standard deviation. By the very nature of a Least Squares adjustment it is assured that the mean of the total set of residuals will be zero. If the quality control procedures, employed to ascertain whether or not the combined set of orbital models provided a best fit to the observation data, included a $\chi^2$ test on the variance of the observation residuals, the assumption of a unit standard deviation is known to be justified. The remaining assumption to be checked is that the residuals are normally distributed. A test of the validity of this assumption can be performed by fitting the residuals to a normal distribution function by a Least Squares adjustment. The goodness of fit of the residuals to this normal distribution function with zero mean and unit standard deviation can then be checked by the application of a $\chi^2$ test.

However, there is little value in going to such great lengths to determine whether or not the residuals are normally distributed. It is more advantageous to restrict the statistical tests to be applied to those that are least affected by the non-normality of the distribution. Reference 9 states on page 173 that the Student's t and variance ratio (or F-ratio) tests can be used with some confidence even if the distributions concerned are quite non-normal. Therefore these two tests are used exclusively throughout this test plan, and the assumption of normally distributed residuals accepted without question.

Since the set of all residuals from the simultaneous multiple orbit reduction are normally distributed (or near normally distributed) with zero mean and unit standard deviation, then all subsets of these residuals chosen at random should have the same distribution. A random choice of subsets infers that the magnitude of the residuals is not to be used as a basis for selection, but since the residuals are assumed to be independent, subsets based on common physical characteristics of the original observations should have the same mean and standard deviation as the total set. If a subset of the residuals, chosen on the basis of some common feature in the original observations, is found to have a mean or standard deviation that is significantly different from the mean or standard deviation of the total residual set, a systematic error correlated with, if not caused by, the common characteristic of this subset should be suspected.

It is impossible to separate the residuals into all of the physically distinguishable subsets. A great deal of judgement must be employed in order to discover the possible sources of systematic errors without initiating a residual sorting project that is too ambitious. A very important consideration is the likelihood of finding a systematic error versus the manhours required to segregate the residuals that belong to that subset. The most practical approach appears to be the separation of residuals based on characteristics that are used in the identification of the residual, and are therefore readily available. Further separation into subsets that are not so easily distinguishable would be avoided unless there is some reason to believe that some systematic error has gone undetected.

## 2.2. Propogation of Random Errors

If all systematic errors have been reduced to an insignificant level, accuracy and precision are equivalent. The accuracy of a computed parameter will then be given by its variance, or more precisely the covariance matrix associated with all parameters computed in the adjustment. The covariance matrix of the computed parameters is a function of not only the random errors of observation but of the geometric distribution of the observations as well. Therefore it is essential that the expected accuracy of ground point locations be computed so that accuracy checks will be made against this figure. If it is found that the actual positions of ground points are within the error bounds established by the computed covariance matrix, then an extimate of the accuracy of any ground point can be obtained by direct computation. Knowing the accuracy of a computed ground point location is a desirable capability even if this accuracy does not meet expectations.

In the adjustment process certain parameters which express the position of the vehicle as a function of time will be computed from the various observation data. Because there are random errors in the observations (it is assumed that all systematic errors were removed from the observations before they were used in the adjustment), there will be errors in the computed positions which can be predicted by standard error propogation techniques. When the expected value of the random errors in two exposure stations are found by a rigorous error propogation, the covariance between the positions of these stations is also obtained. This set of covariances is important because it reflects the relative error between two stations, while the two station covariance matrices show the absolute error in the pair. For example, if a point on the ground is to be intersected from two stations that have little or no correlation, the variance in the intersected point will contain the sum of the variances in the two exposure stations. If. however. the two stations were highly correlated, the variance in the intersected point would not be much larger than the individual variance of either exposure station. Very unrealistic results can be obtained if large correlations between exposure stations are ignored.

In the orbit adjustment the orientation angles of each exposure were considered to be known perfectly from the stellar exposure and were therefore treated as error free. In most cases this assumption was justified because the orientation angles were determined from stellar exposures to a precision far greater than any of the other observed quantities. However, when these exposures are used to intersect points on the ground, the errors in orientation contribute to the errors in the intersected point and should not be neglected. Since the orientation matrix of each frame is obtained from a Least Squares adjustment of the stellar data, the covariance matrix associated with the orientation is readily available and should be employed in any error propogation to computed points on the ground. The resulting propogation of random error in the absence of systematic errors will yield precision estimates associated with the computed ground point position which can be tested by direct checking of the known position. Thus the removal of systematic errors can be checked and if found to be complete, the precision values can be interpreted as accuracy.

## 3. TEST PROCEDURE

The following paragraphs outline a test procedure to be used in assessing the accuracy obtained from a simultaneous adjustment of data from more than one photographic satellite mission. The procedure with minor modifications can be applied to the results from either the TRACE-D or the RECAP Program. This test is aimed at determining the accuracy of a point on the ground that was positioned by photogrammetric intersection from two or more exposure stations whose positions have been obtained from the orbit adjustment. The accuracy of this ground point is necessarily poorer than any of the orbital positions used in its computation, because additional errors will be introduced by the intersection process.

This test can be used to compare the accuracies obtained from TRACE with those of RECAP only under certain circumstances. For instance the same gravitational model must be used in both programs if the results are to be compared and interpreted to be differences in the two programs. Furthermore, the rejection criterion used to eliminate bad observations must be made an comparable as possible for the two programs. Since RECAP rejects observations which give rise to residuals larger than some multiple of the standard deviation of all residuals, and since this is the more statistically acceptable criterion, a final iteration of TRACE must be made on which the rejection level is made to coincide with the particular multiple of the standard deviation of all residuals. Even so there will be differences in the results of the two programs because of the way in which the air drag model is applied, the inclusion or exclusion of the lesser perturbing forces such as third-body effects or radiation pressure, and the means by which the adjustment is performed.

While the procedure given in the following paragraphs is sufficiently general to test the accuracy of any multiple orbit reduction, some of the details would necessarily change if at some future time more areas of geodetic control points become accessible or more missions are available. Furthermore, future data might carry with it more identifying information which would allow more sources of systematic error to be isolated. For example control points might also be identified by local datum, map series, etc., and ground tracking data (as used in RECAP) might carry along the elevation angle to aid in analyzing the effects of residual refraction.

### 3.1. Detection and Removal of Systematic Errors.

As described in Section 2 systematic errors will be detected by dividing the observation residuals into subsets based on some physical characteristic and comparing the mean and standard deviation of each subset to the equivalent statistics for the full set of residuals. It is assumed that the observation residuals are available on a magnetic tape or some other storage device in the order in which they were computed and that each residual is accompanied by certain auxiliary data such as would be required in order to obtain the partial derviatives of each observation equation with respect to the vehicle position. If these data are not a part of the residual record, only a minor program modification will be required to obtain them since they are available in the computer at the time that the observation residuals are formed and output. If only the positions of station and vehicle are included in the residual record, subroutines used in the formation of the system Normal Equations may be used to recompute the required derviatives.

3.1.1. Procedure for Range Residual Analysis

1. Obtain the set of residuals from all observations of Mission No. 1 from Radar Station No. 1. Compute the mean of this sample, given by

$$\bar{v} = \frac{1}{n} \sum_{i=1}^{n} v_i \qquad (1)$$

and compute the statistic

$$t = \frac{\bar{v}\sqrt{n}}{\sigma} \qquad (2)$$

where $\sigma$ is the standard deviation associated with the total adjustment. If the absolute value of t is greater than the value given in Table 1 for the appropriate sample size, the hypothesis that the mean of this subset of residuals is equal to zero must be rejected at the 10% level of significance. More simply stated this means that the probability that a systematic error is affecting this subset of observations is

The failure of this "Student's t-Test" is to be recorded

ble 3a or 3b depending on whether the residuals from the

or TRACE adjustment are being analyzed.

2. Compute the variance of the residual sample acquired for

above test using the formula

$$s^2 = \frac{1}{n} \sum_{i=1}^{n} v_i^2 \qquad (3)$$

e hypothesis of a zero mean was not rejected in the previous

If this hypothesis was rejected, the more involved com-

ion

$$s^2 = \sum_{i=1}^{n} \frac{(v_i - \bar{v})^2}{n - 1} \qquad (4)$$

be used. Next compute the statistic

$$F = \frac{s^2}{\sigma^2} \qquad (5)$$

does not fall between the values of $F_{0.95}$ and $F_{0.05}$ given in

2, the hypothesis that this subset of residuals has the same

nce as the total residual set must be rejected at the 10%

. This failure of the sample variance to satisfy an F-ratio

should be recorded in Table 4a if the residuals are from a

adjustment or Table 4b if a TRACE-D reduction produced them.

3. Repeat (1) and (2) using the residuals from observations from Radar Station No. 1 of Mission Nos. 2, 3, 4, 5, and 6.

4. Repeat (1) through (3) using the residuals from all observations made from Radar Stations Nos. 2, 3, 4, and 5.

3.1.2. Procedure for Tracking Camera Residual Analysis

1. Obtain the residuals from all observations of Mission No. 1 from Tracking Camera No. 1. Compute the mean of the right ascention residuals and the mean of the declination residuals separately, each from a formula equivalent to (1), and then compute the "t" statistics associated with each of these means from formula (2). Compare the absolute value of $t_\alpha$ and $t_\delta$ with the appropriate entries in Table 1 and record in Table 5a or 5b, which ever is applicable, the failure or either value of t to satisfy the t-test criterion.

2. Next compute the variances in the right ascension and declination residuals using formula (3) or (4) depending on whether or not the t-test was satisfied in the previous step. For each of these variances compute the F-ratio given by formula (5) and use these quantities in performing an F-ratio test analogous to that described in (2). Record the failure of either to satisfy the criterion of Table 2 by checking the appropriate box in either Table 6a or 6b.

3. Repeat (1) and (2) using residuals from observations made by Tracking Camera No. 1 of Mission Nos. 2, 3, 4, 5, and 6.

4. Repeat (1) through (3) using the residuals from all observations made from Tracking Camera Nos. 2, 3, ..... etc.

### 3.1.3. Procedure for Control Point Residual Analysis

Each photogrammetric control point will have associated with it five different residuals -- the residuals from the two photo coordinates and the residuals of the three coordinates of the control point since these control point coordinates were considered to be observed variables. In view of the fact that these residuals fall into two distinct types, they will be considered separately.

1. Obtain the set of residuals from all photo coordinates from photograph No. 1 of Mission No. 1. Form the mean of these residuals without separating by x and y-coordinates, and then compute t. Equations (1) and (2) can be used. If the absolute value of t is greater than the value given in Table 1, record the number of this frame for future reference in a list of frames that have failed to satisfy the Student's t-test.

2. Compute the variance of this residual sample using equation (3) or (4), whichever is appropriate on the basis of the above t-Test. Next form the F-ratio; equation (5), and check to see that it falls within the acceptable limits specified by Table 2. If it does not, record the Frame No. in a list of frames that have failed the F-ratio test.

3. Repeat (1) and (2) for each photograph from Mission No. 1. Enter the total number of frames which failed to satisfy either test in Table 9a or 9b.

4. Repeat (1) through (3) for each mission.

5. Obtain the set of all control point coordinate residuals observed by mission No. 1 which were transformed to the World Geodetic System from major datum No. 1. Form the mean of the X-coordinate residuals, the mean of the Y-coordinate residuals, and the mean of the Z-coordinate residuals. Then compute the associated t statistics and compare each of the three to the values given in Table 1. List the Control Points that fail to satisfy the t Test and identify them by latitude and longitude if this information is available.

6. Form the variances associated with each of the three means. Again the formula used for variance computation will depend on whether or not the mean of this residual subset was found to be significantly different from zero. Form the F-ratios from the computed variances and list any failures of these ratios to fall within the acceptable limits given in Table 2.

7. Repeat (5) and (6) for each Mission. Record in Table 7a or 7b the Number of Control Points from Datum No. 1 whose coordinates failed to satisfy either the t-test or the F-ratio test.

8. Repeat (5) through (7) for each Major Geodetic Datum.

3.1.4. Relative Geometry Point Residual Analysis

1. Obtain the set of all residuals from photographic coordinates of Relative Geometry Point No. 1 obtained from Mission No. 1. This may be one, two, or three pairs of residuals from successive photographs, it may include residual pairs from photos made on later passes of the same satellite over the same area, or there may be no residuals due

to the complete lack of photo coverage of this point by Mission No. 1. If there is no photo coverage of this Relative Geometry Point by Mission No. 1, go on to Mission No. 2, 3, 4, 5, or 6; whichever is the first mission on which this RGP was observed. Form the mean of these residuals and compute the t statistic. Form a list of all RGP's that give rise to residuals whose mean value fails to satisfy the t Test.

2. Compute the variance of this residual sample, again using the equation indicated by the results of the above t-test. Compute the F-ratio associated with this variance and form a similar list of RGP's whose residual variance is different from unity as indicated by the F-ratio test of Table 2.

3. Repeat (1) and (2) using all other missions on which this point was observed. In the event that all observation residuals assocaited with this RGP from all missions fail to satisfy the Student's t-Test, or the variances from all missions fail to satisfy the F-ratio test, or both; this RGP should not be included in the final relative geometry point residual statistics. See below.

4. Repeat (1) through (3) for each relative geometry point. Enter in Table 10a or 10b, depending on which adjustment program was used, the total number of RGP's observed on each mission which failed to satisfy either of the tests of (1) or (2). Note, however, the exception stated in (3) and adjust the results accordingly.

3.1.5. Treatment of Mission Dependent Systematic Errors

Tables 3a through 8a and/or 3b through 8b provide a comprehensive summary of the results obtained from the individual t and F-ratio tests for the various kinds of residuals from each mission. Because the tests were based on a 10% confidence level, approximately 10% of the tests conducted are expected statistically to result in failures. However, if for a particular mission more than 10% of the Student's t Tests conducted resulted in failures, the mean of the residuals from this particular category is probably not zero. Likewise the occurrence of a greater than 10% failure rate in the F-ratio tests conducted on residuals associated with a particular mission is evidence of a variance different from unity for that mission. Either of these circumstances points to the likelihood of a systematic error which must be compensated for.

1. If the mean of the residuals associated with a particular mission is significantly different from zero in more than 10% of the cases investigated, this mission is probably affected by one or more systematic errors. Factors which could cause this type of error are: A constant time bias in one or more of the orientation angles, an error in the calibrated values of focal length or principal point coordinates, or a timing error. The possibility of a timing error can be checked by examining the time biases for the various tracking stations that observed this mission. If each of the time bias parameters which were computed in the multi-mission adjustment are of about the same magnitude, there is probably a constant error

### 3.1.7. Treatment of Tracking Station Systematic Errors

In the adjustment program, whether it be RECAP or TRACE-D. the coordinates of each tracking station have been corrected by allowing a set of station shift parameters. Furthermore, the RECAP Program solves for a time bias parameter for all observations from each tracking station of each mission. In spite of these attempts to remove systmatic errors from the ground tracking data, a bias due to a certain tracking site may be indicated in the results of the t and F-ratio tests recorded in Tables 3a, 4a, 5a, and 6a in the case of the RECAP Program evaluation, or in Tables 3b, 4b, 5b, and 6b from TRACE-D. The number of such tests conducted on the residuals from a particular station will be less than ten so it will be difficult to determine whether or not a single failure is significant. Again the magnitude by which the single test was failed must be taken into consideration before a decision can be reached.

1. If the mean value of the residuals in observations made from a particular tracking station is different from zero, a systematic error is indicated. In the case of a radar station there is little that can be done to determine the exact nature of the error beyond verifying that the position of the station was allowed to adjust without unreasonable restriction. There will be little lost in the way of useful information if all observations from this station are dropped from the adjustment. More can be said about the nature of the residuals from a tracking camera, because they have been separated into the directions of topocentric right ascension and declination.

in the vehicle clock, or more precisely, in the relationship between the vehicle time and the external time standard.

If time does not appear to be a problem, the other factors mentioned above must be checked by performing a number of photogrammetric resections in which the calibrated values of focal length, principal, point coordinates, and angular orientation are treated as adjustable parameters rather than known values. The condition equations to be employed are given in Appendix B along with a description of how existing subroutines can be used to form a large portion of the required resection program. The mean value of the parameter corrections obtained from at least 20 frames, well distributed in time and space over the entire mission, should be accepted as a bonafide correction to the calibrated value if the associated standard deviations compare favorably with those obtained for the calibration. Observations should then be corrected and the multiple orbit reduction repeated.

2. If the variance of the residuals associated with a particular mission is greater than unity, the nature of the indicated systematic error is best determined by a transformation of the residuals to a new coordinate system -- the Intrack, Crosstrack, and Radial system. This transformation, applied only to the photogrammetric control point residuals, is accomplished through the formulas given in Appendix B.

If the transformed residuals are plotted as a function of time, they may exhibit a periodicity that is characteristic of a particular perturbing force. Multiple frequency effects may be present which would require that a complete harmonic analysis of the transformed residuals be employed. More likely is the occurrence of one or two dominant frequencies which can be compensated by rerunning the multi-mission adjustment with additional periodic parameters in the empirical form for the orbital elements of this mission. Periodic variations in the radial residuals indicate the need for periodic parameters in Eccentricity and Mean Anonaly. Periodic variations in the Crosstrack residuals can be compensated by periodic parameters in Inclination and Longitude of the Ascending Node. Periodic parameters are required in Eccentricity, Mean Anomaly, and Argument of Perigee to compensate for periodic variations in Intrack residuals.

Another systematic error that may be discovered from the residual plot is a discontinuity in the orbit resulting from an inconsistancy in the correlation of the vehicle clock to the external time standard. There may be other causes for such a discontinuity such as the force applied due to the ejection of a package from the vehicle or the sudden change in atmospheric density which accompanies a large solar flare. When a discontinuity is detected, its cause is immaterial unless more information can be obtained. The removal of this type of systematic error requires a readjustment of the data, treating observations before and after the discontinuity as if they were made from different satellites. Two sets of orbital parameters are then required to describe the path of the vehicle on this single mission, although the number of parameters in each set can probably be reduced because of the shorter time span to be covered.

If, on the other hand, the trnasformed residuals appear to be randomly distributed in time, the suspected systematic error may not exist. The "a priori" estimate of the photo-coordinate measurement variances may have been too small causing a larger variance in residuals that have been weighted by this factor. Such an assumption can certainly be made if the ground tracking data residuals for this mission do not also have a variance larger than unity and/or if the mean of the control point observation residuals was not significantly different from zero. If these conditions exist, the value of $s^2$ computed in this test may be substituted for the "a priori" variance estimate used in the weighting of the DAFF photo measurements from this mission and the multiple orbit reduction rerun.

3. If the variance of the residuals associated with a particular mission is less than unity, there is probably an error in the weighting factor applied to the observations of that mission. If the observations are in fact significantly more precise than they were expected to be, then this increased precision belongs in the covariance matrix of the parameters rather than in the variance of the observation residuals. In the adjustment of a homogeneous set of observations the final variance can be used as a scaling factor applied to the covariance matrix of the adjusted parameters and no further computation is necessary.

This method was used by the DOI-4 Program. In the case at hand however, only a subset of the residuals are so distorted. It is better, therefore, to re-examine the "a priori" variance assigned to the control point measurements from this mission, particularly if analysis of the ground tracking residuals did not verify the apparent mission related bias.

3.1.6. Treatment of Datum Dependent Systematic Errors

Since all tracking stations were considered to be free points, independent of any geodetic datum, the only residuals that might reflect datum biases are those from photogrammetric control point observations. The results of the statistical tests applied to these residuals are summarized in Tables 7a and 8a or 7b and 8b, depending on which of the adjustment programs is being evaluated. Again a 10% failure rate is expected, but in this case fewer than ten tests were conducted on residuals associated with a particular datum. If only one failure occurred, the magnitude of the departure must be considered in deciding whether or not there is sufficient evidence of a datum related systematic error to justify the time and manpower required to perform the further testing described in the following paragraphs.

1. If the mean value of the residuals associated with a particular major geodetic datum is significantly different from zero, a systematic error is indicated which may be due to the transformation

of points on that datum to the World Geodetic System. Since translation of the datum origin was to be included in the parameter set of the multi-mission adjustment as accomplished by either RECAP or TRACE-D, a non-zero mean for these residuals is not likely. Therefore, the computed corrections to the datum origin should be examined to establish that the datum shift adjustment did in fact take place. The next step should then be to check the variances that were applied in advance to the datum shift parameters. It would seem that these parameters were unrealistically restricted by the assignment of too large a weight. This weighting factor should be relaxed and the adjustment repeated.

2. A systematic error associated with control points from a particular geodetic datum should be suspected if the variance of the residuals from observations of those points is significantly larger than one. The errors within any major datum, with the possible exception (at the present time) of the Nan King Datum, should be quite small in comparison to the random errors of observation. However, there are map series within a given major datum that may be systematically distorted with respect to the rest of the maps on the same datum. Since there has been no effort in the past to classify control points by map series, it would be difficult to subdivide the residuals on this basis. If it is possible to further segregate the residuals according to map series, the offending series can be eliminated

from the adjustment. However, it may be impossible or impractical to determine at this stage which control points came from a certain series. The residuals from this datum should then be grouped according to latitude and longitude in order to determine in what manner they may be systematically displaced within the datum.

If one geographic area is the source of the largest residuals and if this area is covered by a single map series, it would be advisable to try to locate another independent map series which covers the same area. New values for the control point coordinates could then be obtained for use in a repeat of the multiple orbit reduction. It is more likely, however, that themap series used was the best one, or the only one available at the time of collecting the data. If such is still the case, the contaminated observations should be omitted from the data set and a new adjustment performed.

Another situation which should be evident from the geographic grouping of residuals is a radial distortion of all points in the datum. In this case observations with the largest residuals would arise from points near the edges of the datum. Errors arising from this situation could be corrected by a new transformation formula which would include a scale factor in the relationship of the particular datum to the World Geodetic System. Lacking such a formula it is probably sufficient to drop all control points on this datum that fall beyond a certain maximum radial distance from the datum origin and then readjust.

If no systematicity can be detected in these residuals, the weighting factors applied to the observations may be unrealistically large. These weights, applied to the observations, should not be confused with the "a priori" weights assigned to parameters such as those discussed in section (1). Incorrect weighting of a group of parameters causes a systematic error by forcing other parameters to compensate for errors in the restricted set. Errors in the weights applied to observations may cause a systematic error by forcing the model parameters to compensate for errors in the observations, if the set of such incorrectly weighted observations is large. If they are a very small set, the variance associated with their residuals will be too large. In this case $s^2$ should be substituted for the "a priori" variance of this observation set and the adjustment repeated.

3. If the variance in the residuals associated with any one of the geodetic datums is significantly less than unity, the control points from this datum have probably been assigned too large a variance (too small a weight). Perhaps the maps from which these points were scaled were at some time in the past downgraded to "Class B" because they lack the cultural detail required of a "Class A" map. The accuracy however might meet "Class A" standards and the variances assigned to points scaled from them would be incorrectly large due to using the accuracy standards of "Class B". The possibility of this type of weighting errors should be investigated. It is advisable that any new variance be verified before it is used. If this verification is not practical, the computed $s^2$ value may be substituted for the assumed observation variance so long as the mean of the residuals is nearly zero.

If only one set of these residuals have a mean value different from zero, the position of the tracking station should be suspected of being in error in that direction. Since such an error should have been corrected in the adjustment, the computed corrections to the tracking camera position should be checked. It is possible that too stringent a weighting factor was placed on the position of this tracker. If not, and if no other information about the nature of the station bias is available, the observation data from this station can be dropped from the adjustment.

2. A variance larger than unity for the subset of residuals from a particular tracking station may be an indication that the data from that station is affected by a bias. One such bias would result from the failure of the pre-processing programs to completely compensate for known systematic errors such as atmospheric refraction. This type of bias could be detected by plotting the residuals from each pass of the satellite over the station as a function of time. If the residuals increase significantly near the beginning and end of each pass over the tracker, then the error model used in preparing this station's data is subject to question.

However, if no systematicity is apparent in this set of residuals and if the mean value is near zero, the assumed variance in the observations from this station is probably in error. As stated in previous paragraphs the application of a weighting factor based on an "a priori" estimate of the observation variances that is too small will cause the residuals in these observations to have a variance greater than one. If this situation is encountered, $s^2$ may be substituted for the previously assigned observation variance for all data from this station and the multi-mission adjustment repeated.

3. If the variance in the residuals from a given tracking station is significantly less than one, it can be assumed that the variance assigned to the observation data from this station has been chosen too large. At this time the best estimate of the observation variance is $s^2$, which should be used in repeating the adjustment.

### 3.2. Propogation of Random Errors

It is assumed at this point that all systematic errors which arise from the major sources of bias have been reduced to an insignificant level. The only errors affecting the vehicle positions obtained from the multiple orbit reduction are the random errors of measurement and/or recording, and those systematic errors which do not cause detectable biases in the subsets of residuals discussed in the last section. In this section the systematic errors will be assumed nonexistant unless inconsistancies are found in the propogation of random errors which indicate that this assumption is not justified. Under this assumption the random errors in the exposure station positions due to the orbit determination and the photogrammetry, when propogated into the computed positions of check points, give the expected accuracy, as opposed to precision, of the computed check point position. If the positions of these check points are known with relatively small errors, then the discrepancy between the true and computed position must not be significantly larger than predicted by the statistical analysis of the random errors, if the hypothesis of no systematic errors is to be accepted.

3.2.1. Selection of Check Points

3.2.1.1. Geodetic Control Points

Figure 1* shows the locations of 39 areas in which geodetic control points have been or are to be selected for use as check points. These areas have been chosen because they provide a good geographic distribution and because it is known from previous tests that most of them contain a number of photo-identifiable geodetic control points compatible with the requirements stated below. Ten to twenty check points are to be selected in each area. This number can be reduced as identification becomes more certain -- three points being sufficient if positively identified. If two geodetic control points are closer together than 20 kilometers, they may be counted

---

* The selected areas of geodetic control are presented in Figure 1 on a homalographic projection of the world (interrupted for the continents) which is a part of the Goode Base Map Series published by the Department of Geography, University of Chicago by whose permission it is used in this report.

SELECTED AREAS OF GEODETIC CONTROL

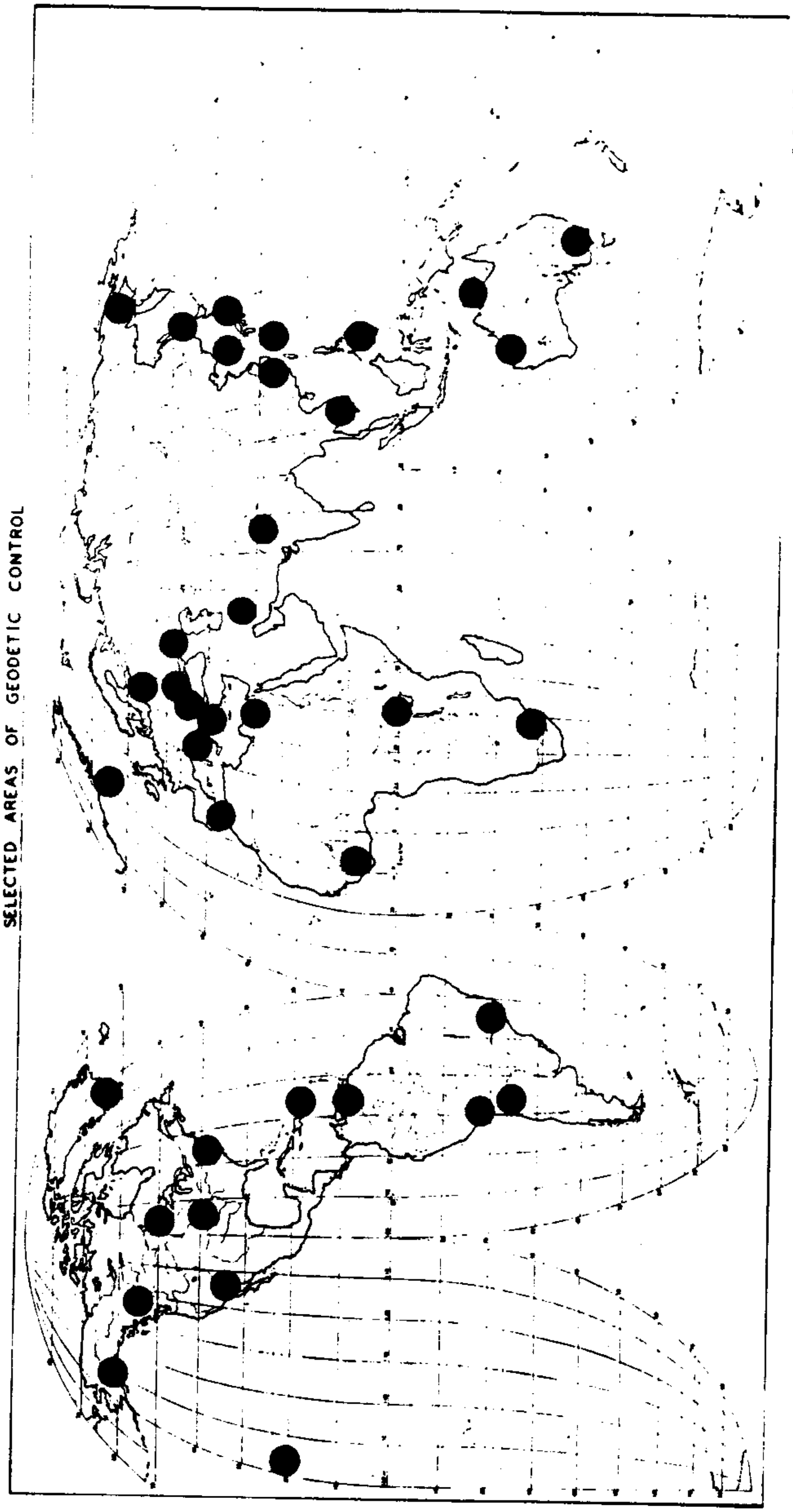

FIGURE 1

as separate points for purposes of obtaining the required number in the area, but some means should be employed of identifying these points as highly correlated.

Each point is to be identified on all photographs on which it can be found. This must include photos from more than one mission or at least from more than one pass on a single mission, although it is not required that all points in a given area of geodetic control fall on any single photograph. In nearly all cases there will be two consecutive photographs on which each point appears, and both of these should be used. This means that most points will be identified on a minimum of four photographs.

3.2.1.2. TRANET Stations

There are approximately 50 TRANET stations that have been located to geodetic accuracy by Doppler observations made from these stations of one or more of the TRANSIT satellites. Less than half of this number is identifiable on the DAFF photography, but if surrounding landmarks are used in transferring the station location from large scale aerial photographs to the DAFF in much the same way in which an area match is accomplished, then approximately half of these stations should be usable as check points. Stations that are of particular interest because of their geographic location are located at Thule, Greenland and MacMurdo Sound in Antarctica, but unfortunately these stations do not appear to be identifiable on the DAFF and no panoramic photography is available of either of these areas. The pacific island stations are also of major interest because they are the farthest away from any control points that may have been used in the adjustment. It appears that many of these island stations will be usable and should be used, even though the exact site of the station can only be approximately located (within 300 or 400 feet).

~~TOP SECRET TRINE~~

~~TOP SECRET RUFF~~

3.2.1.3. Tropo-Scatter Stations

Since there is neither geodetic control nor TRANET station in North-central Siberia, and because of the importance place on the accuracy of point positioning in this area, it is advisable to use as check points any landmark for which coordinates can be obtained regardless of the uncertainties associated with this data. Approximately 31 Tropo-scatter stations are known to exist in this area and the Gauss-Kruger coordinates of most of them are available. These points can be placed within the World Geodetic System with about as much reliability as many geodetic control points in the far east whose coordinates on Nan King Datum are available. However, the identification of most of these stations is subject to serious doubt because several construction sites separated by many hundreds of feet are found in the general area in which the station is known to exist. Each separate site is well defined, but it is impossible to ascertain which of these is the true station.

In such cases all possible station sites should be selected as separate points, and each treated as the true station. If any one of them is found to fall within an acceptable miss-distance of the "true" position of the Tropo-scatter station when intersected from the DAFF photography, then a partial verification of the accuracy of the reduction as it affects this particular area has been obtained. On the other hand a failure to obtain the statistically predicted agreement between computed and "true" positions of the station should not be considered a failure of the orbital model to fit this part of the world unless it is observed that most of the discrepancies in Tropo-scatter stations are in the same direction.



~~TOP SECRET TRINE~~

~~TOP SECRET RUFF~~

HANDLE VIA
~~TALENT-KEYHOLE~~ [redacted]
CONTROL SYSTEMS JOINTLY
~~HANDLE VIA~~
~~TALENT-KEYHOLE~~

3.2.2. Identification of Check Points

The methods used in finding and marking the check points on the DAFF photography will depend to a large extend on the circumstances surrounding the particular point, and should therefore be left up to the operator. It may be better in one case to identify the point on a pan photo first and transfer the identification stereoscopically to the DAFF. However, many points may fall in areas where no panoramic photography is available, or they may fall too near the horizon on the only usable pan photographs. If the operator makes a consciensious effort to maintain the highest degree of accuracy, his judgment as to the most accurate method of identification of the check point should be relied upon.

While precision is generally associated with the measurement phase, identification and marking of the point is probably the largest source of error in the measured coordinates unless the point is measured (without being marked) on a stereo-comparator. It is therefore necessary for the operator who makes the identification and marks the image to make a very careful assessment of the accuracy of his work. This will probably be in the form of an image clarity rating which can later be assigned a weighting factor on the basis of repeatability tests.

3.2.3. Measurement of Check Points

Measurement techniques which produce high standards of accuracy have already been developed in the procedures for evaluation of the individual missions and these techniques should be rigorously followed. In particular all twenty-four shrinkage and fiducial marks should be measured twice and a third reading taken in any case in which the first two do not agree within five microns in the case of contact positives, or twelve microns in 1.8x enlargements. The check point images, or marks indicating the locations of these images, should also be read twice. If the images have been marked by drilling or some other means which provides a small, symmetric target, the taking of a third reading should be based on an agreement criterion similar to that used for fiducial marks, but adjusted in magnitude to account for the greater difficulty in centering on this target. If the images are not marked in advance, a minimum of five independent measurements should be made and an image clarity rating assigned by the comparator operator.

3.2.4. Processing Raw Measurement Data

Certain systematic errors in the photo coordinates must be removed before these data may be used in any photogrammetric intersection program. Among these are radial and tangential lens distortion and film shrinkage. The same corrections that were applied to the operational photography should be used here with two exceptions: (1) Tangential lens distortion corrections should be applied if the calibration data indicates that these are significant, and (2) Film shrinkage should be corrected by applying an eight-parameter transformation. The use of an eight, rather than a six, parameter transformation is predicated on the assumption that the measurements will be made on enlarged positive transparencies rather than contact positives. The tilt introduced in the enlargement process can be compensated by the two extra parameters. If the enlarger has been shown by calibration to be free from these errors, or even if contact prints are used, the eight parameter transformation is preferable because there is some evidence that a tilt has been introduced into some of the frames from at least one of the missions by the failure of the camera pressure plate to seat properly.

3.2.5. Intersection of Check Points

Having at hand the corrected photo coordinates of the conjugate images of a check point, the positions of the camera stations from which the exposures were made, and the orientation of each exposure; the computed position of the check point can be obtained by the standard photogrammetric intersection technique. The procedure given here will differ from the classical intersection because the geocentric position of each exposure station as well as the orientation angles of each exposure are treated as observed variables with covariances known between the coordinates of one exposure station and those of all others on which the point is imaged.

3.2.5.1. Covariance Matrix of Exposure Station Positions

The exposure stations from which the check points are to be intersected can be obtained from the ephemeris generated by the orbit adjustment program. However, the covariance matrix output with the exposure station coordinates does not include covariance between these coordinates and those of any other station. There is instead a better source for this data. On the final iteration of either of the orbital programs the coefficient matrix of the Normal Equations is inverted in order to obtain the covariance matrix associated with the computed orbital parameters. RECAP forms the full covariance matrix which accounts for the correlation between the parameters of one mission and those of another. No such between-missions covariances can be obtained from TRACE without a program modification, but these terms are expected to be small and can probably be neglected. However, the more general case will be treated here and the covariance terms can be assumed to be zero if they are not available.

The matrix of the partial derivatives of the positions of all camera stations to be used in the intersection with respect to the orbital parameters of all missions can be formed by using the appropriate subroutine(s) of the adjustment program (Subroutine PARPAT in the case of the RECAP Program). This is the P matrix in the formulas given in the third part of Appendix B. If the covariance matrix of the orbital parameters from all missions is pre-multiplied by P and post-multiplied by $P^T$, the resulting product is the covariance matrix of the exposure stations from which the check point is to be intersected. This matrix will be required in the weighting of the observed variables used in the intersection.

3.2.5.2. Covariance Matrix of Orientation Angles

The orientation matrix of each photograph was computed by Program OREAXE before the multi-mission adjustment was started. In the course of the orbit adjustment the orientations of all exposures were treated as known, because the inclusion of these quantities either as observed variables or as parameters would so complicate the calculations as to render them completely impractical. This assumption of no error in the roll and yaw angles of the terrain camera is justified because these angles have been found to have a standard deviation of 5 seconds of arc which is well within the noise level of the system. The error-free assumption is nearly valid for the pitch angle since the 20 arc seconds standard deviation that characterizes this quantity contributes an error of about 100 feet on the ground. However, in order to be completely aware of all errors affecting the final results of the check point intersections, the errors in orientation angles of each photograph should be propogated into the accuracy estimate of the check point.

One of the options of the OREAXE Program, known as ORESUM, enables the user to obtain the covariance matrix of the elements of orientation of the stellar exposure. This covariance matrix must be scaled by the measurement variance (also output under this option) converted to inches squared. This transforms the units of the covariance matrix to radians-squared.

Since the orientation of each DAFF exposure is independently determined from the corresponding stellar frame, there will be no correlations between the orientation angles of one frame and those of any other. Hence the covariance matrix of the observed orientation angles of all frames used in intersecting a given check point will be quasi-diagonal, consisting of 3 x 3 submatrices.

3.2.5.3. Covariance Matrix of Photo Coordinates

In the process of selecting, identifying, and measuring the check point images as described in sections 3.2.1. through 3.2.3., estimates of the accuracy of the image coordinates were obtained.

These accuracy estimates are to be put in the form of variances and used as the diagonal elements of the photo coordinate covariance matrix. Since each measurement should be independent, the covariance matrix of these measurements will be diagonal so no covariances need be deduced.

3.2.5.4. Propogation of Errors to Check Point Positions

There are only three parameters to be corrected in the intersection of a check point -- the geocentric coordinates of the check point. This initial approximation of these coordinates can be obtained by scaling the position from a small scale map or the true position can be used because the final coordinates will not depend on the starting values, except that a solution cannot be obtained if starting values of the parameters are too far away from the true values for corrections to be linear. Section 3 of Appendix B gives a detailed description of the intersection process and all of the required formulation.

The propogation of random errors of observation to the intersected check point in order to obtain an estimate of the accuracy of its computed position requires that the observations be weighted according to the covariance matrices discussed in sections 3.2.5.1. through 3.2.5.3. The formation of the weight matrix, or more precisely the inverse of the weight matrix is given by equation (34-B). The covariance matrix of the computed check point position, obtained from inverting the Normal Equation coefficient matrix, gives the predicted accuracy of this position. However, because covariance will exist between the coordinates, this matrix is difficult to interpret. The next section shows how this matrix can be turned into a set of numbers whose significance is much more easily realized.

## 3.3. Analysis of Results

Having computed the position and variance-covariance matrix of the check point, accuracy statements can be made and checked against the true check point position in order to determine whether or not all systematic errors in the orbital positions have been reduced below the random error noise level. Equation (50-B) is the expression for an ellipsoid of constant probability centered on the computed check point position. Note that from the form of this equation the axes of the ellipsoid will not be parallel to the coordinate axes unless all off-diagonal terms in the variance-covariance matrix are zero. Analogous to the probability statements generally made concerning univariate normal distributions, i.e., there is 90% probability that the true value is less than 1.645 times the standard deviation away from the computed mean, there is a 90% probability that the true point will be inside the ellipsoidal surface defined by setting S equal to 6.25.

If the true coordinates of the check point are entered in equation (50-B) in place of the variable coordinates X,Y, Z, then the resulting value of S should be less than 6.25 for approximately

90% of the points tested. If it is not, and the coordinates assumed to be the true position are reliable, then some contamination by systematic errors must still remain. The procedures of section 3.2 should be rechecked to be certain that they have been completed accurately. The next step would be an extensive program of testing further subsets of the residuals by the same methods used in section 3.2. What these subsets might be will depend to a large extent on what categories the residuals can be separated into most easily, unless there is some evidence that indicates that a particular source of error is responsible.

If approximately 90% of the check points do fall within the 90% probability ellipsoid, and if the 10% failures appear to be randomly distributed over the areas used, then the intersection equations of Section 3 of Appendix B can be used to compute the coordinates of unknown points with a high degree of confidence that the resulting covariance matrix will be a reliable accuracy estimator. However, this covariance matrix must be transformed into statistics that are more easily interpretable in terms of the system requirements. The first step will be the rotation of the Geocentric coordinate covariance matrix into a local coordinate system. This rotation, described in Appendix B, allows the elevation variance to be treated separately from the horizontal variances. This is a desirable separation because the horizontal accuracy is expected to be much better than the elevation accuracy.

If the elevation accuracy is extracted from the covariance matrix, the remaining 2 x 2 will be the projection on the horizontal plane of the error ellipsoid. The 90% assurance level for elevation is obtained immediately by multiplying the square root of its variance by 1.645. The horizontal accuracy must be stated in terms of circular error which requires more manipulation. First the eigenvalues of the covariance matrix -- the 2 x 2 associated with the horizontal position must be calculated. A subroutine given in Appendix C can be used for this purpose. The eigenvalues are important because they are the semi-axes of the error ellipse in the horizontal plane. The square root of the ratio of the smaller to the larger eigenvalue is now used as argument for obtaining a value C from Table 11. The radius of the 90% assurance circle is then given by the product of C and the square root of the larger eigenvalue.

The radius of the 90% assurance circle is a single number which characterizes the horizontal accuracy of a particular terrain point. This statistic can be computed for every check point and will probably be found to vary with latitude and longitude. Sufficient check point areas have been employed so that a map of the world showing contours of equal 90% probability circle radii can be constructed. It would then be possible to tell at a glance the horizontal accuracy that can be expected from DAFF intersections of terrain points in any area of the world.

## 4. CONCLUSIONS

Although the complete removal of systematic errors is a nearly impossible task, the reduction of systematic effects on the accuracy of orbital positions to an insignificant level should be accomplishable within the procedures outlined in this test and evaluation plan. Having so reduced the systematic errors, the statistical equations generally employed to obtain estimates of precision may be used in predicting the accuracy of computed results.

With a series of check point areas which are well distributed over the land suffaces of the earth, statistical statements of expected accuracy can be verified. If the accepted geodetic positions of these check points are correct, the accuracy verification will indicate whether or not residual systematic errors remain, and if they do not the computed precision can be accepted as a reliable means of predicting the accuracy of any future point locations obtained from this material.

## 5. RECOMMENDATIONS

It is recommended that this test and evaluation be applied to the adjusted DAFF Missions as soon as these are available. While time spent in the detection and removal of systematic errors by the methods outlined in Section 3.1 does not produce improvements that are readily noticeable, the greatest emphasis should be placed on this part of the project, since the subsequent steps are so highly dependent on it.

There is the distinct possibility that the results of this evaluation will show that while accuracy of ground point location from the DAFF photography is higher than predicted by the more pessimistic critics of the system, it does not meet the tolerances set for it. Even if the DAFF photography used with the best data reduction system does meet present day tolerances, these tolerances will soon be tightened and a more stringent requirement will initiate a program of increasing the accuracy, either by adding more and better control or launching another series of satellites or both. It is therefore recommended that the ultimate accuracy of the existing material be investigated by simulating the effects on the point location accuracy of such new inputs as: more control, more accurate control, a better distributation of control, etc. Such a study will determine whether it is worthwhile to concentrate on the material at hand or supplement this material by further satellite missions. As a byproduct of this study the optimum amount, accuracy, and distribution of control to be employed will be determined. If the improvement of control to be used with the existing material is indicated as the better approach to meeting geodetic accuracy requirements, a plan for control enhancement will be easily formulated from the data obtained from this simulation study.

REFERENCES


1. Anderle, R.J., Error Analysis of the Navy Doppler Tracking System Applied to Geodetic Operations, U.S. Naval Weapons Lab Technical Memo No. K-14/65, January, 1965.

2. Arkin, Herbert and Raymond R. Colton, Tables for Statisticians, Barnes and Noble, Inc., New York, N.Y., 1950.

3. Brown, Duane C., A Treatment of Analytical Photogrammetry with Emphasis on Ballistic Camera Applications. R.C.A. Data Reduction Technical Report No. 39. 1957.

4. Cooley, William W. and Paul R. Lohnes. Multivariate Procedures for the Behavioral Sciences, John Wiley & Sons, Inc., New York, N.Y.. 1950.

5. Crow, Edwin L., Frances A. Davis, and Margaret W. Maxfield, Statistics Manual, Dover Publications, Inc., New York, N.Y.. 1960.

6. Davies, Owen L. (ed.) Statistical Methods in Research and Production, Hafner Publishing Company, New York, N.Y., 1957.

7. Gooding, R.H. "On the Accuracy of Satellite Position Determination", Planetary Space Science. Vol. 12. 1964.

8. Guier, W.H., R.R. Newton, and G.C. Weiffenbach, Analysis of the Observational Contributions to the Errors of the Navy Satellite Doppler Geodetic System, Applied Physics Laboratory Report TG-653, January, 1965.

9. Hamilton, Walter Clark, Statistics in Physical Science, the Ronald Press Company, New York, N.Y., 1964.

10. Izsak, Imre G., Differential Orbit Improvement with the Use of Rotated Residuals, S.A.O. Special Report No. 73.

11. Lucas, James R., "Differentiation of the Orientation Matrix by Matrix Multipliers", Photogrammetric Engineering, Vol. XXIX, No. 4, July, 1963.

12. Veis, G. and C.H. Moore, "Smithsonian Astrophsical Observatory Differential Orbit Improvement Program", J.P.L. Seminar Proceedings.

# APPENDIX A

## TABLES

TABLES (Continued)

TRACE-D Program Evaluation (Continued)

TABLE 1

Ten Per Cent Level of Student's t Distribution

Values of $t_{n-1,0.10}$ such that there is a 10% probability that $|t| > t_{n-1,0.10}$

| n - 1 | $t_{n-1,0.10}$ |
|---|---|
| 5 | 2.02 |
| 6 | 1.94 |
| 7 | 1.90 |
| 8 | 1.86 |
| 9 | 1.83 |
| 10 | 1.81 |
| 15 | 1.75 |
| 20 | 1.72 |
| 25 | 1.71 |
| 30 | 1.70 |
| 40 | 1.68 |
| 50 | 1.68 |
| 60 | 1.67 |
| 100 | 1.66 |
| 200 | 1.65 |
| 500 | 1.65 |
| 1000 | 1.65 |
| Infinity | 1.64 |

TABLE 2

Ten Per Cent Level of the F Distribution

Values of $F_{0.95}$ and $F_{0.05}$ such that the probability that $F_{0.95} < F < F_{0.05}$ is 10%.

| Degrees of Freedom for $s^2$ | Degrees of Freedom for $\sigma^2$ | | | | | |
|---|---|---|---|---|---|---|
| | 100 | | 500 | | Infinity | |
| | $F_{0.95}$ | $F_{0.05}$ | $F_{0.95}$ | $F_{0.05}$ | $F_{0.95}$ | $F_{0.05}$ |
| 5 | 0.435 | 4.40 | 0.444 | 4.37 | 0.453 | 4.36 |
| 6 | 0.456 | 3.71 | 0.472 | 3.68 | 0.479 | 3.67 |
| 7 | 0.476 | 3.28 | 0.493 | 3.24 | 0.498 | 3.23 |
| 8 | 0.493 | 2.98 | 0.510 | 2.94 | 0.516 | 2.93 |
| 9 | 0.508 | 2.76 | 0.526 | 2.72 | 0.532 | 2.71 |
| 10 | 0.521 | 2.59 | 0.541 | 2.55 | 0.547 | 2.54 |
| 12 | 0.540 | 2.35 | 0.562 | 2.31 | 0.572 | 2.30 |
| 14 | 0.559 | 2.19 | 0.582 | 2.14 | 0.592 | 2.13 |
| 16 | 0.572 | 2.17 | 0.599 | 2.02 | 0.610 | 2.01 |
| 20 | 0.595 | 1.90 | 0.625 | 1.85 | 0.637 | 1.84 |
| 30 | 0.637 | 1.69 | 0.671 | 1.64 | 0.685 | 1.62 |
| 40 | 0.663 | 1.59 | 0.705 | 1.53 | 0.715 | 1.51 |
| 50 | 0.676 | 1.52 | 0.725 | 1.46 | 0.741 | 1.44 |
| 75 | 0.705 | 1.43 | 0.758 | 1.36 | 0.782 | 1.34 |
| 100 | 0.720 | 1.39 | 0.782 | 1.30 | 0.806 | 1.28 |
| 200 | | | 0.820 | 1.22 | 0.855 | 1.19 |
| 500 | | | 0.862 | 1.15 | 0.900 | 1.13 |
| Infinity | | | | | 1.000 | 1.00 |





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY

TABLE 3-a

Failure of the Means of Radar Residuals to Satisfy

the Student's t-Test at the 10% Level

| Mission No. | Radar Stations #1 | #2 | #3 | #4 | #5 | Total |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| Total | | | | | | |

TABLE 4-a

Failure of the Variances of Radar Residuals to Satisfy

the F-ratio Test at the 10% Level

| Mission No. | Radar Stations #1 | #2 | #3 | #4 | #5 | Total |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| Total | | | | | | |

TABLE 5-a

Failure of the Means of Tracking Camera Residuals to

Satisfy the Student's t-Test at the 10% Level

Right Ascension Residuals

| Mission | Tracking Camera Stations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

Declination Residuals

| Mission | Tracking Camera Stations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

TABLE 6-a

Failure of the Variances of Tracking Camera Residuals to

Satisfy the F-ratio Test at the 10% Level

Right Ascension Residuals

| Mission | Tracking Camera Stations | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

Declination Residuals

| Mission | Tracking Camera Stations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

~~TOP SECRET RUFF~~

TABLE 7-a

Failure of the Means of Control Point Residuals to Satisfy
the Student's t Test at the 10% Level

X-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Y-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Z-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

~~TOP SECRET RUFF~~

A-HANDLE VIA
~~TALENT-KEYHOLE~~
[illegible] SYSTEM ON[illegible]

TABLE 8-a

Failure of the Variances of Control Point Residuals to Satisfy the F-ratio Test at the 10% Level

X-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Y-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Z-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

TABLE 9-a

Failure of the Means and Variances of the Photo Coordinate Residuals from Control Points to Satisfy the Student's t and F-ratio Tests Respectively at the 10% Level

| Mission No. | Student's t Test (No. of frames) | F-ratio Test (No. of frames) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

TABLE 10-a

Failure of the Means and Variances of the Residuals from Relative Geometry Points to Satisfy the Student's t and F-ratio Tests Respectively at the 10% Level

| Mission No. | Student's t Test (No. of frames) | F-ratio Test (No. of frames) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

TABLE 3-b

Failure of the Means of Radar Residuals to Satisfy
the Student's t-Test at the 10% Level

| Mission No. | Radar Stations | | | | | Total |
|---|---|---|---|---|---|---|
| | #1 | #2 | #3 | #4 | #5 | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| Total | | | | | | |

TABLE 4-b

Failure of the Variances of Radar Residuals to Satisfy
the F-ratio Test at the 10% Level

| Mission No. | Radar Stations | | | | | Total |
|---|---|---|---|---|---|---|
| | #1 | #2 | #3 | #4 | #5 | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | | | | | | |
| 6 | | | | | | |
| Total | | | | | | |

TABLE 5-b

Failure of the Means of Tracking Camera Residuals to Satisfy the Student's t-Test at the 10% Level

Right Ascension Residuals

| Mission | Tracking Camera Stations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

Declination Residuals

| Mission | Tracking Camera Stations | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

TRACE-D Program Evaluation (Con't)

TABLE 6-b

Failure of the Variances of Tracking Camera Residuals to Satisfy the F-ratio Test at the 10% Level

Right Ascension Residuals

| Mission No. | Tracking Camera Stations #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

Declination Residuals

| Mission No. | Tracking Camera Stations #1 | #2 | #3 | #4 | #5 | #6 | #7 | #8 | #9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | | | | | | | | | | |
| 6 | | | | | | | | | | |
| Total | | | | | | | | | | |

TABLE 7-b

Failure of the Means of Control Point Residuals to Satisfy the Student's t Test at the 10% Level

X-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Y-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Z-Coordinate (Geocentric)

| Mission | Major Datum No. | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |


HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY
~~TOP SECRET RUFF~~

TOP SECRET RUFF

TABLE 8-b

Failure of the Variances of Control Point Residuals to Satisfy the F-ratio Test at the 10% Level

X-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Y-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

Z-Coordinate (Geocentric)

| Mission No. | Major Datum No. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 2 | | | | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| Total | | | | | | | | | | | |

TOP SECRET RUFF



HANDLE VIA
TALENT-KEYHOLE
CONTROL SYSTEM

TABLE 9-b

Failure of the Means and variances of the Photo Coordinate Residuals from Control Points to Satisfy the Student's t and F-ratio Tests Respectively at the 10% Level

| Mission No. | Student's t Test (No. of frames) | F-ratio Test (No. of frames) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

TABLE 10-b

Failure of the Means and Variances of the Residuals from Relative Geometry Points to Satisfy the Student's t and F-ratio Tests Respectively at the 10% Level

| Mission No. | Student's t Test (No. of frames) | F-ratio Test (No. of frames) |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |



~~TOP SECRET RUFF~~

HANDLE VIA
~~TALENT KEYHOLE~~
CONTROL SYSTEM ONLY

| P \ C | 0.0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5000 | 0.67449 | 0.68199 | 0.70585 | 0.74993 | 0.80785 | 0.87042 | 0.93365 | 0.99621 | 1.05769 | 1.11807 | 1.17741 |
| 0.7500 | 1.15035 | 1.15473 | 1.16825 | 1.19246 | 1.23100 | 1.28534 | 1.35143 | 1.42471 | 1.50231 | 1.58271 | 1.66511 |
| 0.9000 | 1.64485 | 1.64791 | 1.65731 | 1.67383 | 1.69918 | 1.73708 | 1.79153 | 1.86253 | 1.94761 | 2.04236 | 2.14597 |
| 0.9500 | 1.95996 | 1.96253 | 1.97041 | 1.98420 | 2.00514 | 2.03586 | 2.08130 | 2.14598 | 2.23029 | 2.33180 | 2.44775 |
| 0.9750 | 2.24140 | 2.24365 | 2.25053 | 2.26255 | 2.28073 | 2.30707 | 2.34581 | 2.40356 | 2.48494 | 2.58999 | 2.71620 |
| 0.9900 | 2.57583 | 2.57778 | 2.58377 | 2.59421 | 2.60995 | 2.63257 | 2.66533 | 2.71515 | 2.79069 | 2.89743 | 3.03485 |
| 0.9950 | 2.80703 | 2.80883 | 2.81432 | 2.83289 | 2.83820 | 2.88859 | 2.88859 | 2.93347 | 3.00431 | 3.11073 | 3.25525 |
| 0.9975 | 3.02334 | 3.02500 | 3.03010 | 3.03898 | 3.05234 | 3.07144 | 3.09871 | 3.13969 | 3.20586 | 3.31099 | 3.46164 |
| 0.9990 | 3.29053 | 3.29206 | 3.29673 | 3.30489 | 3.31715 | 3.33464 | 3.35949 | 3.39647 | 3.45698 | 3.55939 | 3.71692 |

TABLE 11: Circular Errors

The entries in this table give values of K such that the probability that a random point falls within a circle of radius $K\sigma_1$ is given by P. The argument C is the ratio of the two standard deviations $\sigma_1$ and $\sigma_2$ where $\sigma_1$ is the larger.

Example:

If $\sigma_1 = 5$ and $\sigma_2 = 2$, then $C = 2/5 = 0.4$ and K associated with the 99% probability level is 2.60995. Therefore $r = K\sigma_1 = 5(2.60995) = 13.05$.





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY

# APPENDIX B

## EQUATIONS AND FORMULAS

### 1. Resection Equations.

Let the position vector of the camera station be denoted $(X_c\ Y_c\ Z_c)^T$ and the position of each ground point $(X_i\ Y_i\ Z_i)^T$ where $i = 1, 2, \ldots n$. These vectors are both in the geocentric coordinate system with the X-axis in the Greenwich Meridian. The Colinearity Condition Equations are

$$x_i - x_p + f\left(\frac{u_i}{w_i}\right) = \varepsilon_{x_i} \qquad (1\text{-B})$$

$$y_i - y_p + f\left(\frac{v_i}{w_i}\right) = \varepsilon_{y_i}$$

where the coordinates subscripted i are refer to the image of ground point i, those subscripted p are the principal point coordinates, f is the terrain camera focal length, and

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} X_c - X_i \\ Y_c - Y_i \\ Z_c - Z_i \end{bmatrix} \qquad (2\text{-B})$$

in which the $m_{ij}$ are the elements of the orientation matrix M of the terrain photograph.

The linearised form of the condition equations will be given by

$$Av + B\delta = \varepsilon \qquad (3\text{-B})$$

where, because in this application the photo coordinates, the control point positions, and the camera station positions are all treated as observed variables, the matrices and vectors involved will be

$$A = \begin{bmatrix} I & 0 & \cdots & 0 & \frac{\partial(\varepsilon x_1 \ \varepsilon y_1)}{\partial(X_1 \ Y_1 \ Z_1)} & 0 & \cdots & 0 & \frac{\partial(\varepsilon x_1 \ \varepsilon y_1)}{\partial(X_c \ Y_c \ Z_c)} \\ 0 & I & \cdots & 0 & 0 & \frac{\partial(\varepsilon x_2 \ \varepsilon y_2)}{\partial(X_2 \ Y_2 \ Z_2)} & \cdots & 0 & \frac{\partial(\varepsilon x_2 \ \varepsilon y_2)}{\partial(X_c \ Y_c \ Z_c)} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 & 0 & \cdots & \frac{\partial(x_n \ y_n)}{\partial(X_n \ Y_n \ Z_n)} & \frac{\partial(x_n \ y_n)}{\partial(X_c \ Y_c \ Z_c)} \end{bmatrix} \qquad (4\text{-B})$$

$$B = \begin{bmatrix} 1 & 0 & u_1/w_1 & \frac{\partial(\varepsilon x_1 \ \varepsilon y_1)}{\partial(\omega \ \theta \ \kappa)} \\ 0 & 1 & v_1/w_1 & \\ 1 & 0 & u_2/w_2 & \frac{\partial(\varepsilon x_2 \ \varepsilon y_2)}{\partial(\omega \ \theta \ \kappa)} \\ 0 & 1 & v_2/w_2 & \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & u_n/w_n & \frac{\partial(\varepsilon x_n \ \varepsilon y_n)}{\partial(\omega \ \theta \ \kappa)} \\ 0 & 1 & v_n/w_n & \end{bmatrix} \qquad (5\text{-B})$$

$$v^T = \left[\Delta x_1 \ \Delta y_1 \ \ldots \ \Delta x_n \ \Delta y_n \ \Delta X_1 \ \Delta Y_1 \ \Delta Z_1 \ \ldots \ \Delta X_n \ \Delta Y_n \ \Delta Z_n \ \Delta X_c \ \Delta Y_c \ \Delta Z_c\right) \qquad (6\text{-B})$$

$$\delta^T = \left[\Delta x_\rho \ \Delta y_\rho \ \Delta f \ \Delta\omega \ \Delta\theta \ \Delta\kappa\right] \qquad (7\text{-B})$$

$$\varepsilon^T = \left[\varepsilon x_1 \ \varepsilon y_1 \ \varepsilon x_2 \ \varepsilon y_2 \ \ldots \ \varepsilon x_n \ \varepsilon y_n\right] \qquad (8\text{-B})$$

Let $\sigma^{o}_{xy_i}$ be the 2 x 2 covariance matrix of the photo coordinate measurements of image point i, $\sigma^{o}_{XYZ_i}$ be the 3 x 3 covariance matrix of the coordinates of the ith control point, and $\sigma^{o}_{XYZ_c}$ the 3 x 3 covariance matrix of the camera station coordinates. The covariance matrix associated with the observed variables will be given by

$$\sigma^{o} = \begin{bmatrix} \sigma^{o}_{xy_1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \sigma^{o}_{xy_2} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma^{o}_{xy_n} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & \sigma^{o}_{XYZ_1} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & \sigma^{o}_{XYZ_2} & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & \sigma^{o}_{XYZ_n} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \sigma^{o}_{XYZ_c} \end{bmatrix} \qquad (14\text{-B})$$

The Normal Equation will be

$$B^{T} (A\sigma^{o} A^{T})^{-1} B\delta = B^{T}(A\sigma^{o} A^{T})^{-1} \varepsilon \qquad (15\text{-B})$$





HANDLE VIA
TALENT-KEYHOLE
CONTROL SYSTEM ONLY

in which the indicated partial derivatives can be written as

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(X_c \ Y_c \ Z_c)} = \frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(u_i \ v_i \ w_i)} \begin{bmatrix} M \end{bmatrix} \qquad (9\text{-B})$$

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(X_i \ Y_i \ Z_i)} = - \frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(X_c \ Y_c \ Z_c)} \qquad (10\text{-B})$$

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(\omega \ \theta \ \varkappa)} = \frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(u_i \ v_i \ w_i)} \cdot \frac{\partial(u_i \ v_i \ w_i)}{\partial(\omega \ \theta \ \varkappa)} \qquad (11\text{-B})$$

where

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(u_i \ v_i \ w_i)} = f \begin{bmatrix} \frac{1}{w_i} & 0 & -u_i/w_i^2 \\ 0 & \frac{1}{w_i} & -v_i/w_i^2 \end{bmatrix} \qquad (12\text{-B})$$

$$\frac{\partial(u_i \ v_i \ w_i)}{\partial(\omega \ \theta \ \varkappa)} = \begin{bmatrix} \frac{w_i \ m_{12}}{(m_{12}^2 + m_{22}^2)^{\frac{1}{2}}} & -v_i \ m_{32} - w_i \ m_{22} & v_i \\ \frac{w_i \ m_{22}}{(m_{12}^2 + m_{22}^2)^{\frac{1}{2}}} & -u_i \ m_{32} + w_i \ m_{12} & -u_i \\ \frac{-u_i m_{12} - v_i m_{22}}{(m_{12}^2 + m_{22}^2)^{\frac{1}{2}}} & u_i \ m_{22} - v_i \ m_{12} & 0 \end{bmatrix} \qquad (13\text{-B})$$

are the factors entering the earlier partial derivatives.

Note that the A matrix is a 2n x (2n + 3n + 3) and the covariance matrix $\overset{o}{\sigma}$ is square and of order 5n + 3. The matrix product $W' = A\sigma^{o}A^{T}$, which is a 2n x 2n, can be formed directly rather than storing the A matrix. It is easily verified that

$$W'_{ii} = \overset{o}{\sigma}_{xy_i} + \left[\frac{\partial(\varepsilon x_i \;\; \varepsilon y_i)}{\partial(X_i \;\; Y_i \;\; Z_i)}\right] \overset{o}{\sigma}_{XYZ_i} \left[\frac{\partial(\varepsilon x_i \;\; \varepsilon y_i)}{\partial(X_i \;\; Y_i \;\; Z_i)}\right]^T + \left[\frac{\partial(\varepsilon x_i \;\; \varepsilon y_i)}{\partial(X_c \;\; Y_c \;\; Z_c)}\right] \overset{o}{\sigma}_{XYZ_c} \left[\frac{\partial(\varepsilon x_i \;\; \varepsilon y_i)}{\partial(X_c \;\; Y_c \;\; Z_c)}\right]^T \qquad (16\text{-B})$$

for the diagonal elements, and

$$W'_{ij} = \left[\frac{\partial(\varepsilon x_i \;\; \varepsilon y_i)}{\partial(X_c \;\; Y_c \;\; Z_c)}\right] \overset{o}{\sigma}_{XYZ_c} \left[\frac{\partial(\varepsilon x_j \;\; \varepsilon y_j)}{\partial(X_c \;\; Y_c \;\; Z_c)}\right]^T \qquad (17\text{-B})$$

for all off-diagonal elements.

The Normal Equations (15-B) are solved iteratively for the parameter correction vector $\delta$. The inverse of the coefficient matrix $B^T(A\sigma^{o} A^T)^{-1}B$ will then be the covariance matrix of the adjusted parameters. This matrix should provide a good indication whether the adjusted values for the calibration parameters are expected to be superior to those previously used.

## 2. Residual Transformation Equations

The transformation of the residuals in photographic coordinates of control points to the equivalent residuals in the Radial, Intrack, and Crosstrack position of the camera station may be accomplished by the equation

$$\begin{bmatrix} \Delta \text{ Radial} \\ \Delta \text{ Intrack} \\ \Delta \text{ Crosstrack} \end{bmatrix} = Q \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta x_2 \\ \Delta y_2 \\ \vdots \\ \Delta x_n \\ \Delta y_n \end{bmatrix} \qquad (18\text{-B})$$

where the $\Delta x_i$ and $\Delta y_i$ are residuals in the photo coordinates of the control points imaged on a single photograph, the transformed residuals Δ Radial, Δ Intrack, and Δ Crosstrack apply to the position of the camera station, and the transformation matrix Q is given by

$$Q = (V^T U^T U V)^{-1} V^T U^T \qquad (19\text{-B})$$

where

$$U = \begin{bmatrix} \dfrac{\partial(\varepsilon x_1 \;\; \varepsilon y_1)}{\partial(X_c \;\; Y_c \;\; Z_c)} \\ \dfrac{\partial(\varepsilon x_2 \;\; \varepsilon y_2)}{\partial(X_c \;\; Y_c \;\; Z_c)} \\ \vdots \\ \dfrac{\partial(\varepsilon x_n \;\; \varepsilon y_n)}{\partial(X_c \;\; Y_c \;\; Z_c)} \end{bmatrix} \qquad (20\text{-B})$$

is the matrix of the partial derivatives of the co-linearity condition equation with respect to the camera station coordinates as given in equation (9-B), and

$$V = \frac{1}{R} \begin{bmatrix} X_c & \frac{\partial X_c}{\partial \omega} & \frac{1}{\sin i} & \frac{\partial X_c}{\partial i} \\ Y_c & \frac{\partial Y_c}{\partial \omega} & \frac{1}{\sin i} & \frac{\partial Y_c}{\partial i} \\ Z_c & \frac{\partial Z_c}{\partial \omega} & \frac{1}{\sin i} & \frac{\partial Z_c}{\partial i} \end{bmatrix} \qquad (21\text{-B})$$

The elements of these matrices can be obtained from the subroutine in the RECAP Program which forms the required derivative, i.e., Subroutine BXYZ for elements of U, and Subroutine PARPAT for the partials required in V. A similar Subroutine is available in the TRACE-D Program for computing the partial derivatives required in the V matrix, but since the colinearity condition equation is not used in this program the elements of U must be obtained from RECAP or from separate computation.

For completeness the partial derivatives of the camera position with respect to the orbital elements, required in the formation of V follow. These formulas can be used if separate programs are to be used rather than incorporating existing subroutines.

$$\frac{\partial X_c}{\partial \omega} = R(-\sin \ell \cos \lambda - \cos \ell \cos i \sin \lambda)$$

$$\frac{\partial Y_c}{\partial \omega} = R(-\sin \ell \sin \lambda + \cos \ell \cos i \cos \lambda) \qquad (22\text{-B})$$

$$\frac{\partial Z_c}{\partial \omega} = R \cos \ell \sin i$$

$$\frac{\partial X_c}{\partial i} = Z_c \sin \lambda$$

$$\frac{\partial Y_c}{\partial i} = - Z_c \cos \qquad (23\text{-B})$$

$$\frac{\partial Z_c}{\partial i} = R \cos i \sin \ell$$

$$R = a(1 - e \cos E) \qquad (24\text{-B})$$

$$\ell = \omega + \nu \qquad (25\text{-B})$$

where

$$E = M + e \sin E \qquad (26\text{-B})$$

$$\sin \nu = \frac{a(1 - e^2)}{R} \sin E$$

$$\cos \nu = \frac{a(\cos E - e)}{R} \qquad (27\text{-B})$$

and a, e, i, $\omega$, $\lambda$, and M are the semi-major axis, eccentricity, inclination, argument of perigee, longitude of the ascending node, and mean anomaly of the orbit respectively.

## 3. Intersection Equations

The intersection equations following the same line of development as the resection equations given in the first section of this appendix. Let the first approximation of position of the ground point (check point) be $(X_g\ Y_g\ Z_g)^T$ and the positions of the camera stations from which the check point was observed are $(X_{ci}\ Y_{ci}\ Z_{ci})^T$ where $i = 1, 2, \ldots n$. The co-linearity condition equations will be

$$\begin{aligned} x_i + f\left(\frac{u_i}{w_i}\right) &= \varepsilon x_i \\ y_i + f\left(\frac{v_i}{w_i}\right) &= \varepsilon y_i \end{aligned} \qquad (28\text{-B})$$

where x and y are the coordinates of the check point image on the ith photograph and

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} m_{11}^i & m_{12}^i & m_{13}^i \\ m_{21}^i & m_{22}^i & m_{23}^i \\ m_{31}^i & m_{32}^i & m_{33}^i \end{bmatrix} \begin{bmatrix} X_{ci} - X_g \\ Y_{ci} - Y_g \\ Z_{ci} - Z_g \end{bmatrix} \qquad (29\text{-B})$$

in which the $m_{jk}^i$ are the elements of the orientation matrix $M_i$ of the ith photograph.

In order to form the weight matrix for the observed variables the covariance matrices of each of the observed quantitites must be obtained. The covariance matrix of the several exposure station positions can be formed from the covariance matrix of the orbital parameters as obtained from the adjustment program. This matrix will have the form

$$\sigma_P = \begin{bmatrix} \sigma_{P_1P_1} & \sigma_{P_1P_2} & \cdots & \sigma_{P_1P_m} \\ \sigma_{P_2P_1} & \sigma_{P_2P_2} & \cdots & \sigma_{P_2P_m} \\ \vdots & \vdots & & \vdots \\ \sigma_{P_mP_1} & \sigma_{P_mP_2} & \cdots & \sigma_{P_mP_m} \end{bmatrix} \qquad (30\text{-B})$$

where $\sigma_{P_iP_i}$ is the covariance matrix of the orbital parameters of the ith mission and $\sigma_{P_iP_j}$ is the matrix which expresses the covariance between the parameters of the ith and jth missions. The propogation of covariance to the exposure stations requires the matrix

$$P = \begin{bmatrix} \frac{\partial(X_{c_1}\ Y_{c_1}\ Z_{c_1})}{\partial P_1} & \frac{\partial(X_{c_1}\ Y_{c_1}\ Z_{c_1})}{\partial P_2} & \cdots & \frac{\partial(X_{c_1}\ Y_{c_1}\ Z_{c_1})}{\partial P_m} \\ \frac{\partial(X_{c_2}\ Y_{c_2}\ Z_{c_2})}{\partial P_1} & \frac{\partial(X_{c_2}\ Y_{c_2}\ Z_{c_2})}{\partial P_2} & \cdots & \frac{\partial(X_{c_2}\ Y_{c_2}\ Z_{c_2})}{\partial P_m} \\ \vdots & \vdots & & \vdots \\ \frac{\partial(X_{cn}\ Y_{cn}\ Z_{cn})}{\partial P_1} & \frac{\partial(X_{cn}\ Y_{cn}\ Z_{cn})}{\partial P_2} & \cdots & \frac{\partial(X_{cn}\ Y_{cn}\ Z_{cn})}{\partial P_m} \end{bmatrix}$$

in which each submatrix is the set of partial derivatives of the particular exposure station with respect to the orbital parameters of a specific mission. It is obvious that an exposure station is a part of only one mission and therefore independent of all others.





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ON.

Hence only one of the indicated submatrices in each row of P will be nonzero. All other submatrices in that row are shown here only to indicate the number of zeros required to fill out the matrix. Specifically, if the first mission is characterized by $p_1$ parameters, the second by $p_2$, and the ith by $p_i$; and if the jth exposure station occurred on the ith mission; then $\partial(X_{cj}\ Y_{cj}\ Z_{cj})/\partial P_i$ is a 3 x $p_i$ matrix of partial derivatives which can be obtained from the subroutine used in forming the system Normal Equations (Subroutine PARPAT in the case of RECAP). The other submatrices in that row, $\partial(X_{cj}\ Y_{cj}\ Z_{cj})/\partial P_k$ for all k except k = i, will be 3 x $p_k$ matrices of all zeros.

The covariance matrix of positions of the set of exposure stations that will be used in intersecting the check point can now be obtained from

$$\sigma^o_{XYZ} = P\sigma_P P^T \qquad (31\text{-B})$$

The covariance matrix associated with the orientation angles of each stellar exposure is obtained from Program OREAXE if the proper option is used. This matrix must be multiplied by a scale factor derived from the measurement variance (output in the form of a standard deviation) in order to convert the units to radians squared:

$$\sigma^o_\alpha = \left(\frac{\sigma_r}{25.4}\right)^2 \sigma^o_x{}'$$

where $\sigma_r$ is the measurement standard deviation, $\overset{o}{\sigma}_\alpha{}'$ is the covariance matrix output from OREAXE and $\overset{o}{\sigma}_\alpha$ is the covariance matrix of the orientation angles as required in forming the weight matrix. Since the orientation of each DAFF photograph is determined from the corresponding stellar frame, there will be no correlation between the orientations of pairs of photos. The required covariance matrix $\overset{o}{\sigma}_{\alpha\delta s}$ of the orientation angles of all frames used in the intersection will therefore be quasidiagonal, consisting of a series of 3 x 3's down the principal diagonal.

The covariance matrix of the image coordinates $\overset{o}{\sigma}_{xy}$ will be diagonal, consisting of the variances in the image coordinates as determined at the time of identification or measurement.

Hence the covariance matrix of the observed variables will be

$$\sigma^o = \begin{bmatrix} \overset{o}{\sigma}_{xy} & 0 & 0 \\ 0 & \overset{o}{\sigma}_{XYZ} & 0 \\ 0 & 0 & \sigma^o{}_{\alpha\delta_s} \end{bmatrix} \qquad (32\text{-}B)$$

and the matrix A of the partial derivatives of the linearized condition equations with respect to the observed variables will be of the form

$$A = \begin{bmatrix} I & A_{XYZ} & A_{\alpha\delta_s} \end{bmatrix} \qquad (33\text{-}B)$$

so that

$$A\sigma^o A^T = \sigma^o_{xy} + A_{XYZ}\, \sigma^o{}_{XYZ}\, A^T_{XYZ} + A_{\alpha\delta s}\, \sigma^o_{\alpha\delta s}\, A^T_{\alpha\delta s} \qquad (34\text{-}B)$$

In this equation

$$A_{XYZ} = \begin{bmatrix} \frac{\partial(\varepsilon x_1 \ \varepsilon y_1)}{\partial(X_{c_1} \ Y_{c_1} \ Z_{c1})} & 0 & \cdots & 0 \\ 0 & \frac{\partial(\varepsilon x_2 \ \varepsilon y_2)}{\partial(X_{c2} \ Y_{c2} \ Z_{c2})} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{\partial(\varepsilon x_n \ \varepsilon y n)}{\partial(X_{cn} \ Y_{cn} \ Z_{cn})} \end{bmatrix} \quad (35\text{-B})$$

is a 2n x 3n in which each submatrix is given by

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(X_{ci} \ Y_{ci} \ Z_{ci})} = f \begin{bmatrix} \frac{1}{w_i} & 0 & -u_i/w_i^2 \\ 0 & \frac{1}{w_i} & -v_i/w_i^2 \end{bmatrix} \begin{bmatrix} M_i \end{bmatrix} \quad (36\text{-B})$$

and

$$A_{\alpha\delta s} = \begin{bmatrix} \frac{\partial(\varepsilon x_1 \ \varepsilon y_1)}{\partial(\alpha_1 \ \delta_1 \ s_1)} & 0 & \cdots & 0 \\ 0 & \frac{\partial(\varepsilon x_2 \ \varepsilon y_2)}{\partial(\alpha_2 \ \delta_2 \ s_2)} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \frac{\partial(\varepsilon x_n \ \varepsilon y_n)}{\partial(\alpha_n \ \delta_n \ s_n)} \end{bmatrix}$$

is also a 2n x 3n in which

$$\frac{\partial(\varepsilon x_i \ \varepsilon y_i)}{\partial(\alpha_i \ \delta_i \ s_i)} \quad f \begin{bmatrix} \frac{1}{w_i} & 0 & -u_i/w_i^2 \\ 0 & \frac{1}{w_i} & -v_i/w_i^2 \end{bmatrix} \frac{\partial(u_i \ v_i \ w_i)}{\partial(\alpha_i \ \delta_i \ s_i)} \quad (38\text{-B})$$

The orientation matrix $M_i$ of the terrain frame is obtained from the stellar camera orientation matrix by the rotation

$$M_i = R\, H_i \qquad (39\text{-B})$$

where R is the matrix which was obtained in the calibtation of this instrument and H is the stellar orientation matrix output from ~~OREAXE~~ in which the elements are

$$\begin{aligned}
h_{11} &= -\cos s \sin \alpha + \sin s \cos \alpha \sin \delta \\
h_{12} &= +\cos s \cos \alpha + \sin s \sin \alpha \sin \delta \\
h_{13} &= -\sin s \cos \delta \\
h_{21} &= -\sin s \sin \alpha - \cos s \cos \alpha \sin \delta \\
h_{22} &= +\sin s \cos \alpha - \cos s \sin \alpha \sin \delta \qquad (40\text{-B}) \\
h_{23} &= +\cos s \cos \delta \\
h_{31} &= +\cos \alpha \cos \delta \\
h_{32} &= +\sin \alpha \cos \delta \\
h_{33} &= +\sin \delta
\end{aligned}$$

Because of the relationship (39-B), equation (29-B) can be written

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = R\, H_i \begin{bmatrix} X_{ci} - X_g \\ Y_{ci} - Y_g \\ Z_{ci} - Z_g \end{bmatrix} \qquad (41\text{-B})$$





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY

Using the matrix differentiation technique described in reference 11 the required partial derivatives can be obtain from

$$\frac{\partial}{\partial s}\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = R\,\frac{\partial H_i}{\partial s}\begin{bmatrix} X_{ci} - X_g \\ Y_{ci} - Y_g \\ Z_{ci} - Z_g \end{bmatrix}$$

$$= R\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} R^T \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \qquad (42\text{-B})$$

$$\frac{\partial}{\partial \delta}\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \frac{1}{(1 - h_{33}^2)^{\frac{1}{2}}} R \begin{bmatrix} 0 & 0 & -h_{13} \\ 0 & 0 & -h_{23} \\ h_{13} & h_{23} & 0 \end{bmatrix} R^T \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \qquad (43\text{-B})$$

$$\frac{\partial}{\partial \alpha}\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = R \begin{bmatrix} 0 & h_{33} & -h_{23} \\ -h_{33} & 0 & h_{13} \\ h_{23} & -h_{13} & 0 \end{bmatrix} R^T \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \qquad (44\text{-B})$$

The matrix B of the partial derivatives of the observation equations with respect to the check point position is given by

$$B = \begin{bmatrix} \dfrac{\partial(\varepsilon x_1 \;\; \varepsilon y_1)}{\partial(X_g \;\; Y_g \;\; Z_g)} \\ \dfrac{\partial(\varepsilon x_2 \;\; \varepsilon y_2)}{\partial(X_g \;\; Y_g \;\; Z_g)} \\ \vdots \\ \dfrac{\partial(\varepsilon x_n \;\; \varepsilon y_n)}{\partial(X_g \;\; Y_g \;\; Z_g)} \end{bmatrix} \qquad (45\text{-B})$$



~~TOP SECRET RUFF~~

HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM

where again

$$\frac{\partial(\varepsilon x_i \ \ \varepsilon y_i)}{\partial(X_g \ Y_g \ Z_g)} = - \frac{\partial(\varepsilon x_i \ \ \varepsilon y_i)}{\partial(X_{ci} \ Y_{ci} \ Z_{ci})} \qquad (46\text{-B})$$

The Normal Equations to be solved are then

$$B^T(A\sigma^o A^T)^{-1} BD = B^T(A\sigma^o A^T)^{-1}\varepsilon \qquad (47\text{-B})$$

in which

$$D = (\Delta X_g \ \ \Delta Y_g \ \ \Delta Z_g)^T, \qquad (48\text{-B})$$

is the vector of parameter corrections, and

$$\varepsilon = (\varepsilon x_1 \ \varepsilon y_1 \ \varepsilon x_2 \ \varepsilon y_2 \ . \ . \ . \ \varepsilon x_n \ \varepsilon y_n)^T, \qquad (49\text{-B})$$

the discrepancy vector, are the only variables not defined in equations (32-B) through (46-B). Solving this equation iteratively until the parameter corrections become negligible will result in the most probable position of the check point, and the inverse of the Normal Equation coefficient matrix.

$$\sigma_g = B^T(A\sigma^o A^T)^{-1}B^{\ -1}$$

will be the covariance matrix associated with this position.





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ONLY

## 4. Error Analysis Equations

The quadratic form

$$\begin{bmatrix} X - X_g & Y - Y_g & Z - Z_g \end{bmatrix} B^T(A\sigma^o A^T)^{-1}B \begin{bmatrix} X - X_g \\ Y - Y_g \\ Z - Z_g \end{bmatrix} = S \qquad (50\text{-}B)$$

where $X_g$, $Y_g$, and $Z_g$ are the computed coordinates of the check point position and $B^T(A\sigma^o A^T)^{-1}B$ is the coefficient matrix of the normal equations employed in the solution for the check point position, is the equation of an ellipsoid of constant probability. That is, if S is set equal to 6.25 the expression (50-B) is the equation of an ellipsoid centered on the point $X_g, Y_g, Z_g$ such that there is a 90% probablility that the true point falls inside the surface. Hence, if X,Y,Z is set equal to the true coordinates of the check point, then the resulting value of S should be less than 6.25 for 90% of the points tested. If this is not the case, then either the assumed true position of the check point is in error or there is still a systematic error affecting the orbital positions.

By transforming the covariance matrix into a local coordinate system, the horizontal and vertical components can be treated separately. This transformation is given by

$$\sigma_L = K\ \sigma_g\ K^T \qquad (51\text{-}B)$$





HANDLE VIA
~~TALENT-KEYHOLE~~
CONTROL SYSTEM ON

Where $\sigma_L$ is the covariance matrix in the local coordinate system, $\sigma_g$ is the covariance matrix in the geocentric coordinate system, and

$$K = \begin{bmatrix} \cos\Phi \cos\lambda & \cos\Phi \sin\lambda & \sin\Phi \\ -\sin\lambda & \cos\lambda & 0 \\ -\sin\Phi \cos\lambda & -\sin\Phi \sin\lambda & \cos\Phi \end{bmatrix} \quad (52\text{-B})$$

in which $\Phi$ is the geodetic latitude, and $\lambda$ is longitude.

The elevation variance $\sigma_h^2$ is the 3,3-element of $\sigma_L$. The 90% assurance interval for elevation will be the square root of this variance multiplied by 1.645.

The horizontal position covariance matrix is the 2 x 2 obtained from deleting the last row and column form $\sigma_L$. The eigen-values of this matrix can be obtained directly from subroutine EIGEN2 given in Appendix C of this report.

The ratio of the square roots of these eigenvalues

$$C \quad \frac{\varepsilon_1}{\varepsilon_2}$$

is used as argument in Table 11 to obtain a value of K such that

$$r = K \quad \varepsilon_2$$

is the radius of the associated probability circle. The units of this radius will be the same as those of the original covariance matrix since the eigenvalues are the result of expressing the error ellipse in a coordinate system where there is no covariance.

# APPENDIX C

## SUBROUTINES

```
      SUBROUTINE CUAD (COEF, ROOT1, ROOT2, ROOTI)
C
C
C     THIS SUBROUTINE COMPUTES THE ROOTS OF THE QUADRATIC EQUATION
C              COEF(1)*X**2 + COEF(2)*X + COEF(3) = 0
C     THE OUTPUT CONSISTS OF THE REAL PARTS OF THE TWO ROOTS (ROOT1 AND
C     ROOT2) AND THE IMAGINARY PART (ROOTI) WHICH APPLIES TO BOTH.
C
      DIMENSION COEF(3)
C
C     COMPUTE THE FIRST TERM
      TERM1 = -COEF(2)/(2.*COEF(1))
C     COMPUTE THE DISCRIMINANT
      DISCR = TERM1**2 - COEF(3)/COEF(1)
      IF (DISCR) 20, 10, 10
C
C     THE ROOTS OF THIS EQUATION ARE REAL
   10 TERM2 = SQRTF(DISCR)
      ROOT1 = TERM1 + TERM2
      ROOT2 = TERM1 - TERM2
      ROOTI = 0.
      GO TO 30
C
C     THE ROOTS OF THIS EQUATION ARE COMPLEX
   20 TERM2 = SQRTF(-DISCR)
      ROOT1 = TERM1
      ROOT2 = TERM1
      ROOTI = TERM2
C
   30 RETURN
      END
```

```
      SUBROUTINE EIGEN2 (A, EIGENV)
C
C
C     THIS SUBROUTINES COMPUTES THE EIGENVALUES OF A SYMMETRIC MATRIX OF
C     THE SECOND ORDER.
C
C     THE INPUT CONSISTS OF THE MATRIX A(2,2) AND--
C     THE OUTPUT EIGENV(2) IS A VECTOR OF THE EIGENVALUES OF A.
C
C     SUBROUTINES REQUIRED BY THIS SUBROUTINE ARE--
C     QUAD (COEF, ROOT1, ROOT2, ROOTI)
C
      DIMENSION A(2,2), COEF(3), EIGENV(2)
C
C     FORM COEFFICIENTS OF THE CHARACTERISTIC EQUATION.
      COEF(1) = 1.
      COEF(2) = -(A(1,1) + A(2,2))
      COEF(3) = A(1,1)*A(2,2) - A(1,2)**2
C     SOLVE THE CHARACTERISTIC EQUATION.
      CALL QUAD (COEF, EIGENV(1), EIGENV(2), ROOTI)
C
   10 RETURN
      END
```