Orbital Mechanics

Version 14.1
Learning Objectives

At the end of this lesson you should be able to:

- Recognize the fundamental laws to get to and stay in orbit
- Correctly apply basic terminology
- Describe the different types of orbits
Orbits: Gravity & Motion

- An orbit is the constrained motion of a body (think satellite) about a common center of mass (think the earth)
- There are two forces acting on a body in space
  - Gravity provides an attractive force between two bodies
  - Motion (velocity) provides a force that tends to separate two bodies
- Put two bodies in space with no motion and they will pull toward one another
Balancing Forces

- Newton’s Law of Universal Gravitation:

\[ F = \frac{G m_1 m_2}{r^2} \]

Where:
- \( G \) = gravitational constant
  - \((6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\)
- \( m_1 \) = mass of object 1
- \( m_2 \) = mass of object 2
- \( r \) = distance between them

- Newton’s 2\(^{nd}\) Law:

\[ F = ma \]

Where:
- \( F \) = Force acting on the body
- \( m \) = mass
- \( a \) = acceleration (change in velocity)
Kepler’s 1st Law

- The orbits of the planets are ellipses with the sun at one focus

- The law also applies to satellites orbiting the earth
Kepler’s 2\textsuperscript{nd} Law

- The line joining the planet to the sun sweeps out equal area in equal times

- This allows us to determine the speed of an orbiting body at various points in the orbit

\begin{align*}
\text{time}_1 & = \text{time}_2 \\
\text{area } A &= \text{area } B
\end{align*}
Defining an Orbit

To specify a satellite's orbit in space, we need to know four things about it:

- **Size** – Semi-Major Axis
- **Shape** – Eccentricity
- **Orientation** – Inclination
- **Location** – Ground Track
Semi-Major Axis = a

- Describes the size of an orbit: half the distance of the major axis
- The larger the “a,” the larger the orbit, the greater the energy

**Perigee** - Closest distance the satellite passes from the primary body
  - Forward velocity of the satellite is at its greatest
  - Gravitational pull from the Earth is strongest point

**Apogee** – Furthest distance the satellite reaches from the primary body
  - Forward velocity of the satellite is at its minimum point
  - Gravitational pull from the Earth is weakest point
Eccentricity - e

- **Eccentricity (e)** defines the elongation of an ellipse and measures the shape of an orbit.
- The value of eccentricity in any closed orbit lies between zero and one.
- The larger the value, the more elliptical the orbit.
Eccentricity – e (con’t)

- Shape of the orbit, described as the type of conic section
  - Circle $e = 0$
  - Ellipse $0 < e < 1$
  - Parabola $e = 1$
  - Hyperbola $e > 1$
Inclination - $i$

- Describe the tilt of the orbit plane with respect to the equatorial plane
Inclination – i (con’t)

- $i = 0^\circ$ (Equatorial)
- $i = 90^\circ$ (Polar)
- $0^\circ < i < 90^\circ$ (Prograde)
- $90^\circ < i < 180^\circ$ (Retrograde)
Satellite Ground Tracks

- Ground tracks are the intersection of the orbit plane with the rotating earth
- Due to the earth’s rotation, the ground track shifts as the satellite revolves around the earth
- The amount of shift depends upon the altitude, eccentricity, and inclination

Orbit to Ground Track Video
The satellite is “fixed” in inertial space – the orbit plane doesn’t move unless acted on by some external force.

But the earth rotates under the satellite

- This makes the satellites ground path move west or east over the surface of the earth
Ground Tracks of Near-Polar Sat
Shuttle Ground Track Example

51.6 Degrees Prograde Inclination
Total Energy in an Orbit

Low Kinetic Energy

Min Velocity

Apogee

Decreasing Velocity

High Kinetic Energy

Perigee

Max Velocity

High Potential Energy

Increasing Velocity

Low Potential Energy

Total Energy = Kinetic Energy + Potential Energy
How Satellites Get to Orbit

- A satellite sitting on a booster is only affected by one of the forces (gravity) since it has no velocity.
- To achieve orbit a satellite must accelerate to a velocity sufficient to have gravity balanced by motion.

\[ \frac{G m_1 m_2}{r^2} = F = \frac{m_2 a}{F} \]
Gravity will cause a projectile to fall 5 meters in one second.

The earth’s curvature is 5 meters for each 8000 meters along the surface (assuming a spherical earth).

To avoid impacting the surface, the projectile must travel at least 8000 meters before it falls 5 meters, i.e., within one second.

Minimum velocity is 8000 meters/sec (17,600 mph) with smooth spherical earth and no atmosphere.
Getting to Orbit

- Launch vehicle accelerates satellite to orbital velocity for minimum-energy sustainable orbit; a low earth orbit (blue).
- To achieve final orbit, satellite fires thrusters to increase velocity (adding energy), which makes it climb and causes orbit to become elliptical (black).
- Thrusters are fired at desired perigee.
- Satellite fires thruster again to adding energy and circularizing orbit (dashed).
Getting to Orbit (con’t)

• Multi-burn transfer:
  - Initial process same as single burn transfer
  - As necessary, satellite fires thruster again, adding more energy and increasing ellipticity
  - As before, thruster fires at desired perigee
  - When desired apogee altitude is reached, thrusters fire to circularize orbit (or reduce ellipticity and raise perigee)
Common Types of Orbits

- Polar Axis
- Polar Orbit
- Geosynchronous Orbit
- Low Earth Orbit
- HEO Orbit
Low Earth Orbit (LEO)

- Satellite deployment
- Surveillance
- Usually circular w/ constant height
- Most periods ~ 90 minutes w/ constant velocity
- Full range of inclinations
- Examples
  - Space Shuttle
  - International Space Station (ISS)
Highly Elliptical Orbit (HEO)

- Navigation (Nav) and Communications (Comms)
- Highly elliptic
- Period is 12 hours (Semi-Synchronous)
- Inclinations between 50 to 70 degrees
- Examples:
  - Molniya
Molniya Orbit (HEO)

The Instantaneous Field of View (FOV)

Molniya Orbit to Ground Track Video
• Comms & Surveillance
• Most circular orbits
• Periods near 24 hrs
  ■ a = 23,000 miles
  ■ i = 0 to 15 degrees

• Geostationary
  ■ Circular, 24 hr period
  ■ i = 0

• Examples:
  ■ MILSTAR
  ■ Satellite TV
Geostationary vs. Geosynchronous

<table>
<thead>
<tr>
<th>Geostationary</th>
<th>Geosynchronous</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = 0$</td>
<td>Period $= 24$ hrs</td>
</tr>
<tr>
<td>$i = 0$</td>
<td></td>
</tr>
</tbody>
</table>

Period $= 24$ hrs

- The Geostationary orbit has a more stringent criteria than a Geosynchronous orbit
Geosynchronous Orbits

- A geostationary orbit that isn’t over the equator
- If the orbit is circular, but inclined, the ground track is a symmetric figure 8

- If the orbit is a 24 hour orbit, but elliptical, the figure 8 is asymmetric
Other Orbits of Interest

- MEO
  - Between ~2000 – 35786km, usually ~20000km
  - Circular
  - GPS, GLONASS, Galileo
- Non-synchronous elliptic orbits
  - Inclinations between 50 to 70 degrees
Repeating Ground Track Example

- Sirius Satellite Radio wanted the minimum number of satellites possible to give 24/7 coverage over the US, Canada, and Central/South America.

- Considerations
  - Polar orbits would require dozens of satellites
  - Geos can’t see into Canada
Perturbations

- **Perturbations** – External forces affect satellite orbits and vary depending on the altitude and inclination

- **Types:**
  - **Earth’s Oblateness** – Gravitational forces not uniform
  - **3rd Bodies** - Moon, Sun, Planets
  - **Solar Wind** - Increasing effects further out in space
  - **Earth’s Electromagnetic Field** – Charged particles affects metal components on the spacecraft
  - **Atmospheric Drag** - Slows satellites down and bleeds off satellite energy
## Dominant Perturbations (con’t)

<table>
<thead>
<tr>
<th>Type of Orbit</th>
<th>Oblateness</th>
<th>3rd Body Effects</th>
<th>Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO</td>
<td>Large</td>
<td>Small</td>
<td>Moderate</td>
</tr>
<tr>
<td>HEO</td>
<td>Moderate</td>
<td>Moderate</td>
<td>Moderate</td>
</tr>
<tr>
<td>GEO</td>
<td>Small</td>
<td>Large</td>
<td>None</td>
</tr>
</tbody>
</table>
Making Orbit Adjustments

- Orbit adjustments are made to:
  - Correct for perturbation effects
  - Nodal changes to collect new targets
  - Optimize collection in case of launch / spacecraft failure

- Orbit Adjustments take significant fuel and can reduce operating life
  - Changes are carefully planned
  - Executed to minimize fuel consumption
Making Orbit Adjustments (con’t)

- Orbit adjustment to change nodal position
  - Westward move
    - $+ \Delta V$ to move to higher altitude
    - Higher altitude, less $V$, longer period
  - At correct node
    - $- \Delta V$ to drop back to geosynchronous orbit
  - Eastward move, reverse process
Orbital Selection Summary

- Orbits are selected to balance:

<table>
<thead>
<tr>
<th>Item</th>
<th>Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution / Sensitivity</td>
<td>Altitude (lower is better)</td>
</tr>
<tr>
<td>Coverage</td>
<td>Trade between number of satellites and altitude</td>
</tr>
<tr>
<td>Access</td>
<td>Trade between number of satellites and altitude</td>
</tr>
<tr>
<td>Cost</td>
<td>Number of satellites and capability</td>
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Summary

- You should now be able to:
  - Recognize the fundamental laws to get to and stay in orbit
  - Correctly apply basic terminology
  - Describe the different types of orbits